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THESIS

COMPUTER DESIGN OF CONTROL SYSTEMS
USING ALGEBRAIC METHODS

by

Jesus E. D'Paola Lozada

December 1969

Thesis
D7235

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Computer Design of Control Systems
Using Algebraic Methods

by

Jesus E. D'Paola Lozada
Lieutenant Commander, Venezuelan Navy

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1969

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[redacted]

Academic Dean

Thesis D7235 c.2

ABSTRACT

Algebraic method techniques are applied to design linear control systems. Computer programs to accomplish the design procedure are presented. Several combinations of feedback and cascade compensated systems using different specifications are studied. Specifications considered include the system damping ratio, undamped natural frequency, bandwidth, steady state error coefficients and root sensitivity.

The method is shown to be applicable to any transfer function and capable of working with any number of parameters.

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I. INTRODUCTION

The design problem can be stated as "the problem of determining a practical set of physical parameters for a system such that a desired performance is obtained." [1]

Many techniques have been devised to accomplish the design of control systems: the Root Locus and the conventional frequency response techniques being the most popular. All of these techniques present the advantage that the designer has a graphical representation of the problem and therefore can visualize the effects that a change in one parameter produces in the others. However, they are time consuming and have the disadvantage of being unable to handle more than one parameter. One way of overcoming this difficulty is by using another graphical technique capable of handling more than one parameter. For this reason Siljak [2] has developed the "parameter plane" method, in which two parameters may be varied at the same time and the solution may be obtained by using the transformation proposed earlier by Mitrovic [3].

The theory and methodology developed for the parameter plane have provided the feasibility of using a pure analytic multiparameter procedure called "algebraic methods," by means of which the system differential equations and the specifications expressed in analytical form can be reduced to a set of algebraic equations. The resultant algebraic equations, however, are nonlinear and in all but the most simple cases solution by hand calculations may be a more difficult problem than the actual problem of design. This difficulty has been greatly overcome in the last decade by the rapid development of digital computers, which have made possible the application of numerical techniques for the solution of nonlinear equations.

In this paper two methods are used to solve the design problem. The first method uses a linearization procedure to solve the resultant nonlinear equations, while with the second method the equations are solved by an iterative method.

II. ALGEBRAIC METHODS

The basis for Mitrovic's approach to the root finding problem [1] was the Cauchy principle of argument which states that "the conformal mapping of a closed contour on the S plane through a mapping function produces a contour on the polar plane, and the encirclements of the origin of the polar plane by the mapped curve can be interpreted in terms of the singularities of the mapping function that are enclosed by the S plane contour." Mitrovic chose, for the values of S in the second quadrant, the transformation of variable:

$$S = W_n \exp(j(\pi/2)+\theta) = -W_n \operatorname{Sin}\theta + jW_n \operatorname{Cos}\theta \qquad (2\text{-}1a)$$

for a mapping function given by the polynomial on the S plane

$$F(s) = \sum_{k=o}^{n} A_k S^k = 0 \qquad (2\text{-}2)$$

If in equation 2-1a Wn is the radial distance from the origin to a particular root (later on this will be called the dominant root) of F(s), and θ the angle formed between the second quadrant radial line and the negative real axis, and also defining cos θ as the damping ratio ζ, then equation 2-1a can be written as

$$S = -\zeta W_n + jW_n(1 - \zeta^2)^{1/2} \qquad (2\text{-}1)$$

Furthermore if 2-1 is substituted in 2-2 it can be shown [2] that the powers of S formed obey the following relations

$$S^k = W_n^k (T_k(-\zeta)) + j[(1 - \zeta^2)^{1/2} U_k(-\zeta)]$$

where

$$T_k(-\zeta) = (-1)^k T_k(\zeta)$$

$$U_k(-\zeta) = (-1)^k U_k(\zeta)$$

and U_k and T_k are Chebyshev functions of the first and second kind respectively and are given by the following recurring relations [2]

$$T_{k+1}(\zeta) = 2\zeta T_k(\zeta) - T_{k-1}(\zeta)$$

$$U_{k+1}(\zeta) = 2\zeta U_k(\zeta) - U_{k-1}(\zeta)$$

with

$$T_o(\zeta) = U_1(\zeta) = 1$$

$$U_o(\zeta) = 0 \text{ and } T_1(\zeta) = \zeta$$

Using these transformations and equating to zero independently real and imaginary parts, equation 2-2 can be written

$$\sum_{k=o}^{n} (-1)^k A_k W_n^k U_{k-1}(\zeta) = 0 \qquad (2\text{-}3)$$

$$\sum_{k=o}^{n} (-1)^k A_k W_n^k U_k(\zeta) = 0 \qquad (2\text{-}4)$$

Equations 2-3 and 2-4 are the basic equations developed by Siljak [2] to be used in the "parameter plane" method of design. They are also the basis, as will be apparent in the following sections, by which a pure analytical design is made. That is, design using pure analytical procedures consists basically of solving equations 2-3 and 2-4 for the unknown parameters of the characteristic equation formed by the system differential equations, reduced to a transfer function in terms of the

Laplace transform (S domain), and the adequate cascade and/or feedback compensators required to satisfy the prescribing specifications. It is obvious that the coefficients A_k in equations 2-3 and 2-4 will be a function of the unknown parameters of the system and those introduced to it by the compensator devices. If these parameters are called r_i (i = 1, 2, ...P) then the coefficients can be written as

$$A_k = G_k(r_i) \quad i = 1, \ldots, p \tag{2-5}$$

Requirement for solution is that the number of equations must equal the number of unknowns. Therefore a set of P equations are required to attempt a solution. The trivial way of finding these equations is by assuming P roots of the characteristic equation of the compensated system and replace each pair of complex ones in 2-3 and 2-4, and each real one in the equation

$$\sum_{k=o}^{n} A_k \sigma^k = 0 \tag{2-6}$$

Another philosophy consists of substituting in the basic equations only the roots whose real parts have the least negative value and dominate the time response of the system (by definition this is the dominant root) and derive additional algebraic relationships from the system performance specifications. In previous work algebraic relationships for specifications such as bandwidth, sensitivity and error coefficients have been found. However, they have the disadvantage that depending on the complexity of the system they lead to a set of algebraic equations whose nonlinearity is such that the solution is extremely difficult and assistance of a digital computer is mandatory.

III. THREE SPECIFICATIONS DESIGN

A. ROOTS AND ERROR COEFFICIENT SPECIFICATIONS

Three common performance specifications are the dominant roots (actually two specifications) and the error coefficient. The cominant roots are a measure of the time response of the system while the error coefficient can be interpreted in terms of the steady state errors (classical interpretation) or as a measure of the moments of the impulse response of the closed loop system function [4]. From this last interpretation, time delay and rise time can also be derived.

The design problem in terms of the three specifications, or in general in terms of any number of specifications, can be broken into three parts:

a. The specifications are interpreted in terms of constraints on the mathematical functions describing system characteristics.

b. A number of systems are determined, each of which satisfies the prescribing specifications; and

c. The "best" system is selected.

If conventional frequency response and root locus methods are used the procedure that the designer follows is to modify and build up the open loop transfer function until the closed loop meets the prescribed specifications (basically trial and error); but if pure analytical procedures are used the problem can be solved in a rather fast and accurate way.

1. General Statement of the Problem

Design a linear control system whose uncompensated open loop transfer function, in terms of Laplace transform, is given by a ratio of two polynomials and whose closed loop must be such that a given set of specifications for root locations and error constant coefficient are satisfied. The prescribed specifications imply that basically three equations with three unknowns must be found in order to attempt a solution; that is, the root specifications lead to a set of two equations and the other equation is derived from the error coefficient specification. The designer therefore must find appropriate compensator devices which, when combined with the system equations in a closed form, meet the specifications. Several combinations of feedback and cascade compensators may be used. However, in this section, a combination of single filter cascade and velocity feedback (tachometer feedback) is used to develop a computer program capable of handling a rather general problem in which the transfer function is assumed formed by the ratio of two polynomials expressed either in its factored form or in powers of the complex variable s. Since in most devices used for control systems, the order of the numerator polynomial is much less than the order of the denominator, it is further assumed that the uncompensated denominator transfer function is at least two orders greater than the uncompensated numerator transfer function. The system block diagram to be analyzed can be arranged as shown in Figure 3-1.

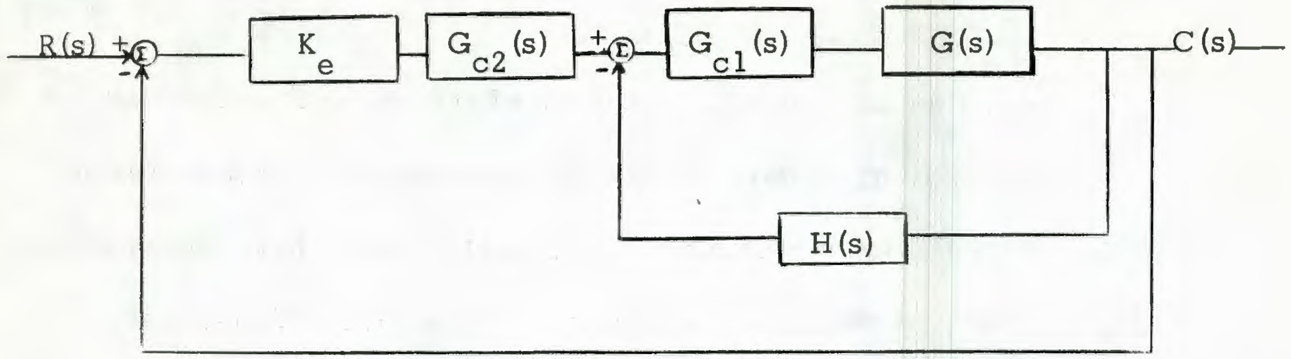


Figure 3-1

Cascade and Feedback Compensation for Roots and Error Coefficient Specifications Design

Considering $G_{c2}(s) = 1$, the system transfer function can be written as:

$$G(s) = K_f \frac{N(s)}{D(s)} = K_f \frac{A_1 S^m + A_2 S^{m-1} + \ldots\ldots + A_{m+1}}{Q_1 S^n + Q_2 S^{n-1} + \ldots\ldots + Q_{n+1}} \quad (3\text{-}1)$$

Likewise the cascade compensator (single filter) can be written as:

$$G_{c1}(s) = \frac{S + Z}{S + P} \quad (3\text{-}2)$$

and the feedback compensator as:

$$H(s) = Kt.S \quad (3\text{-}3)$$

Equations 3-1, 3-2 and 3-3 contain four unknowns, namely Kf, Z, P and Kt, which when combined to form the characteristic equation of the system will be a function of the coefficients, i.e.,

$$A_k = G_k\ (Kf,\ Z,\ P,\ Kt) \quad (3\text{-}4)$$

In fact if the characteristic equation is defined as F(s), then

$$F(s) = 1 + Go\ (S) = 0$$

$$F(s) = 1 + Ge(S).K_e \quad (3\text{-}5)$$

where

$$G_e(s) = \frac{G_{c1}(s)G(s)}{1 + [G_{c1}(s)G(s)]K_t S}$$

$$G_o(s) = K_e G_e(s)$$

Defining the closed loop transfer function as C(s)/R(s), this can be written as:

$$\frac{C(s)}{R(s)} = \frac{G_o(s)}{1 + G_o(s)} = \frac{K_e G_e(s)}{1 + K_e G_e(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_e K_f N(s)(S+Z)}{(S+P)D(s) + K_f K_t SN(s)(S+Z) + K_e K_f N(s)(S+Z)}$$

$$\frac{C(s)}{R(s)} = \frac{P(s)}{F(s)} \qquad (3-6)$$

Expansion of equations 3-5 and 3-6 was needed in order to write the computer program in Appendix D. However, the author considers that no insight, beyond the fact that the coefficients of P(s) and F(s) are functions of the unknown parameters, is gained by writing again such expansion.

If n is the order of the uncompensated denominator polynomial and m the order of the corresponding numerator and if $n \geq m+2$ then the order of the characteristic equation will be n+1; therefore, it can be written as:

$$F(s) = \sum_{k=o}^{n+1} G_k(K_f K_t, Z, P)S^k = 0 \qquad (3-7)$$

From the basic eauations 2-3 and 2-4 and the given root specifications expressed in polar form (equation 2-1) the following equations can be written:

$$\sum_{k=o}^{n+1} (-1)^k G_k(K_f, K_t, Z, P) W_o^{\,k} U_{k-1}(\zeta) = 0 \qquad (3\text{-}8)$$

$$\sum_{k=o}^{n+1} (-1)^k G_k(K_f, K_t, Z, P) W_o^{\,k} U_k(\zeta) = 0 \qquad (3\text{-}9)$$

The error coefficient specification can be expressed in algebraic form in the following way:

let Goc(s) be the open loop transfer function of the compensated system, then

$$G_{oc}(s) = \frac{K_e K_f (A_1 S^{m+1} + (A_1 Z + A_2) S^m + \ldots\ldots\ldots + A_{m+1} Z)}{(Q_1 S^{n+1} (Q_1 P + Q_2) S^n + \ldots + Q_{n+1} P) K_f K_t (A_1 S^{m+2} + \ldots\ldots)}$$

From this expression the three more common types of systems can be defined in terms of the coefficients in the following way:

Type zero system: Q_{n+1} is not equal to zero

Type one system: Q_n is not zero and Q_{n+1} is equal to zero

Type two system: Q_n and Q_{n+1} are both equal to zero

The constant error coefficient for a type zero system can then be written as

$$K_p = \lim_{s \to o} G_{oc}(s) = \frac{K_f A_{m+1} Z K_e}{Q_{n+1} P} \qquad (3\text{-}10)$$

Similar equations can be written for a type one and type two system. However, they will not be considered at this point due to the fact that they are of such a form that a linearization procedure to solve the system of four equations in four unknowns is not possible. On the other hand the problem can still be solved using the iterative procedures of program FISDEP in Appendix D.

The given set of specifications has been used to find equations 3-8, 3-9 and 3-10. They constitute a set of three equations in four unknowns, therefore, another equation is required. This equation can be an auxiliary equation by means of which the designer can force a specific real root of the system to be placed at a determined location, allowing in this form that the given root specification will have at least a real part less than one of the real roots of the system (for a second order uncompensated transfer function this procedure will insure the dominance of the given root specification). Under this consideration the needed fourth equation is:

$$F(s) = \sum_{k=o}^{n+1} G_k(K_f, K_t, Z, P)\, \sigma_x^k \tag{3-11}$$

If the coefficients G_k(Kf, Z, P, Kt) of equations 3-8, 3-9, 3-10 and 3-11 are expanded, the following matrix equation can be written:

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{3-12}$$

where:

$r_1 = K_f K_e Z$

$r_2 = P$

$r_3 = K_f K_t$

$r_4 = K_e K_f + K_f K_t Z$

and d_{ij} are the constant coefficients of r_j in equation i (i = 3-8, .etc)

c_i are the constant terms in equation i (i = 3-8, ...3-11)

Equation 3-12 is a linear algebraic matrix equation of the form $\underline{D} \cdot \underline{r} = \underline{C}$. Solution of this equation is $\underline{r} = \underline{D}^{-1} \cdot \underline{C}$. If the found solution is used to determine the values of the unknown parameters the following two alternatives result:

$$K_{f1} = (1/2K_e)\,(r_4 + (r_4^2 - 4r_1r_3)^{1/2})$$

$$K_{t1} = r_3/K_{f1}$$

$$Z_1 = r_1/K_eK_{f1}$$

$$P = r_2$$

alternative two

$$K_{f2} = (r_4/2K_e)\,(1 + (r_4^2 - 4r_1r_3)^{1/2})$$

$$K_{t2} = r_3/K_{f2}$$

$$Z_2 = r_1/K_eK_{f2}$$

$$P = r_2$$

The proper choice of either alternative depends mainly on the numbers involved and secondly on the designer criterion when both solutions are physically possible. On the other hand it might be possible that neither solution one nor solution two is physically possible or that the system becomes unstable with one or the other solution. In both cases another combination of compensator devices might be used.

All the development has been done so far considering an external loop gain fixed; however, some problems might require trading a variable external gain, K_e, by a fixed transfer function gain, K_f. Obviously if this is the case the above development still holds and the solution of the problem gives as before two alternatives, in which the compensator pole and the tachometer gain have the same numerical value while two values for external loop gain are found at the expense of the compensator zero. The following solutions can be written:

alternative one

$$K_{e1} = (r_4/2K_f)\,(1 + (r_4^{\,2} - 4r_1r_3)^{1/2})$$

$$Z_1 = r_1/K_fK_{e1}$$

$$K_t = r_3/K_f$$

$$P = r_2$$

alternative two

$$K_{e2} = (r_4/2K_f)\,(1 - (r_4^{\,2} - 4r_1r_3)^{1/2})$$

$$Z_2 = r_1/K_fK_{e2}$$

$$K_t = r_3/K_f$$

$$P = r_2$$

Considering $G_{c1} = 1$ and $G_{c2} = \dfrac{S + Z}{S + P}$, a similar development as the foregoing case (G_{c2} (s) = 1) can be done and the problem can again be solved by finding a solution to equation 3-12 with the following definitions:

$$r_1 = K_eK_fZ$$

$$r_2 = P$$

$$r_3 = K_fK_t$$

$$r_4 = K_eK_f + K_fK_tP$$

Unlike the case where the filter is inside the internal loop, a unique solution is obtained in this case. On the other hand whether the external gain or the transfer function gain is the variable, the equations containing the unknown parameters have the same form, that is

$$P = r_2$$

$$K_f = (r_4 - r_3 P)/K_e$$

$$K_t = r_3/K_f$$

$$Z = r_1/K_e K_f$$

A computer program capable of handling all the cases considered was written by the author and is shown in Appendix D.

2. Examples

Several examples will show the feasibility of using the computer program of Appendix D for different uncompensated transfer functions.

Example 3.1

Design a filter compensator and find values for forward and feedback gain which will give the system shown in Figure 3-2 a constant error coefficient of 2.0 and a pair of complex roots located at $\xi = 0.5$ and $W_n = 5.0$. It is also specified that a real root be located at $\sigma_x = -8.0$.

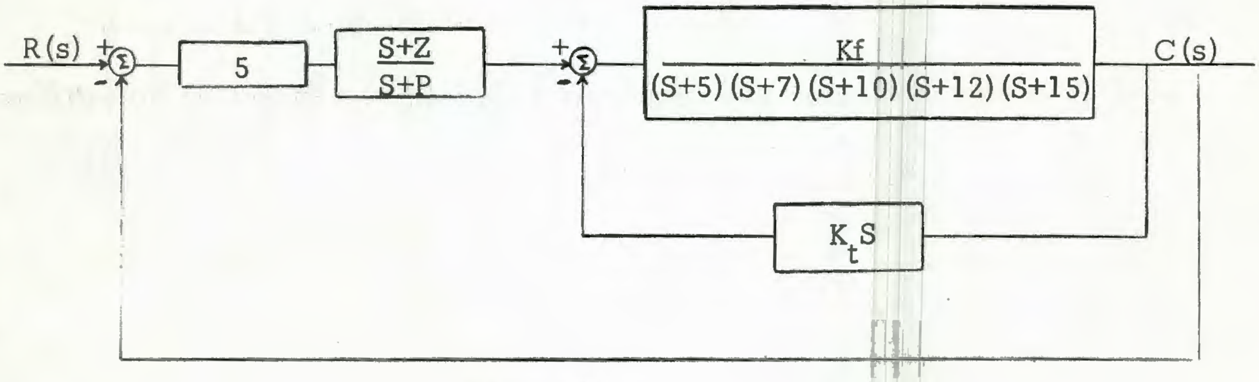


Figure 3-2

Block Diagram for Example 3.1

```
INPUT DATA
----------


SPECIFICATION FOR ZETA IS........................   0.50000E 00

SPECIFICATION FOR NATURAL FREQUENCY IS...........   0.50000E 01

SPECIFICATION FOR REAL ROOT IS..................  -0.80000E 01

SPECIFICATION FOR ERROR COEFFICIENT IS...........   0.20000E 01

SPECIFICATION FOR EXTERNAL LOOP GAIN IS..........   0.50000E 01

COEFFICIENTS OF NUMERATOR POLYNOMIAL IN DESCENDING ORDER
    0.10000E 01

COEFFICIENTS OF DENOMINATOR POLYNOMIAL IN DESCENDING ORDER
    0.10000E 01    0.49000E 02    0.92900E 03    0.84950E 04
    0.37350E 05    0.63000E 05

REAL ZEROS OF UNCOMPENSATED TRANSFER FUNCTION


    0.0

COMPLEX CONJUGATES ZEROS OF UNCOMPENSATED TRANSFER FUNCTION


    0.0            0.0

REAL POLES OF THE UNCOMPENSATED TRANSFER FUNCTION


   -0.50000E 01   -0.70000E 01   -0.10000E 02   -0.12000E 02


   -0.15000E 02

COMPLEX CONJUGATES POLES OF UNCOMPENSATED TRANSFER FUNCTION


    0.0            0.0
```

```
OUTPUT DATA
****** ****

FILTER OUTSIDE INTERNAL LOOP, EXTERNAL GAIN SPECIFIED

FORWARD GAIN..............K=     0.10995E 05
TACHOMETER GAIN..........KT=     0.44516E 00
COMPENSATOR POLE.........PC=     0.14699E 01
COMPENSATOR ZERO.........ZC=     0.33689E 01

COMPUTED ERROR COEFFICIENT IS =     0.20000E 01

COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.27782E 06    0.18007E 06    0.54732E 05

    0.98606E 04    0.10010E 04    0.50470E 02

    0.10000E 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY

   -0.47188E 01    0.0

   -0.80001E 01    0.0

   -0.16376E 02   -0.51197E 01

   -0.16376E 02    0.51197E 01

   -0.25000E 01   -0.43301E 01

   -0.25000E 01    0.43301E 01
```

Example 3-2

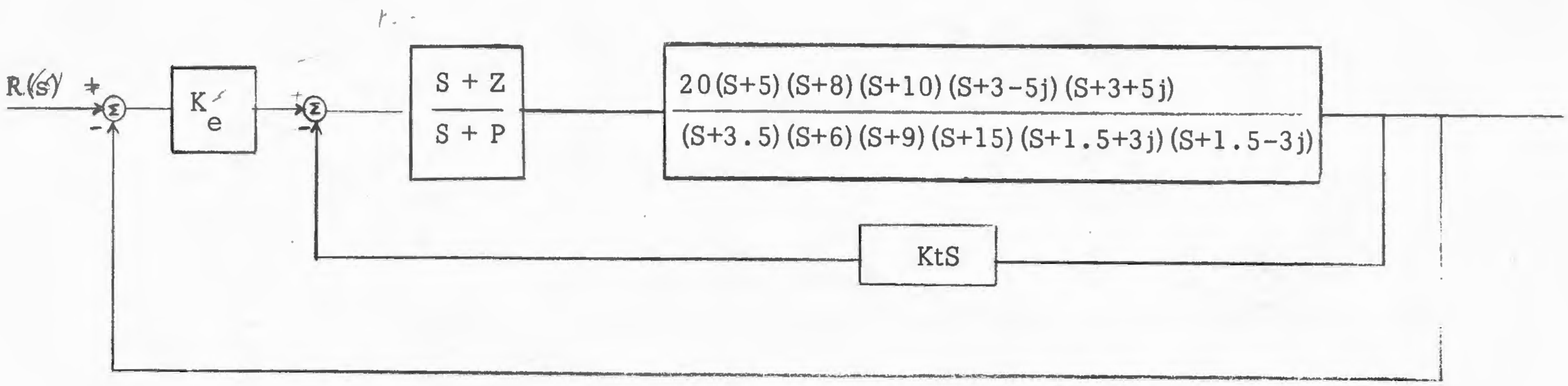

R(s)
+
-
K_e
+
-
$\frac{S + Z}{S + P}$
$\frac{20(S+5)(S+8)(S+10)(S+3-5j)(S+3+5j)}{(S+3.5)(S+6)(S+9)(S+15)(S+1.5+3j)(S+1.5-3j)}$
KtS

Figure 3-3
Block Diagram for Example 3-2

Design a filter compensator and find values for external loop and feedback gains which will give the system shown in Figure 3-3 a constant error coefficient of 5.0 and a pair of complex roots located at $\zeta = 0.5$ and $W_n = 5.0$. It is also specified that a real root be located at $\sigma_x = -10.0$.

```
   DATA
 --------


SPECIFICATION FOR ZETA IS........................   0.50000E 00

SPECIFICATION FOR NATURAL FREQUENCY IS...........   0.50000E 01

SPECIFICATION FOR REAL ROOT IS...................  -0.10000E 02

SPECIFICATION FOR ERROR COEFFICIENT IS...........   0.50000E 01

SPECIFICATION FOR TRANSFER GAIN IS...............   0.20000E 02

COEFFICIENTS OF NUMERATOR POLYNOMIAL IN DESCENDING ORDER
    0.10000E 01    0.29000E 02    0.34200E 03    0.22020E 04
    0.81800E 04    0.13600E 05

COEFFICIENTS OF DENOMINATOR POLYNOMIAL IN DESCENDING ORDER
    0.10000E 01    0.36500E 02    0.49575E 03    0.33154E 04
    0.12514E 05    0.28603E 05    0.31894E 05

REAL ZEROS OF UNCOMPENSATED TRANSFER FUNCTION

   -0.50000E 01   -0.80000E 01   -0.10000E 02

COMPLEX CONJUGATES ZEROS OF UNCOMPENSATED TRANSFER FUNCTION

   -0.30000E 01    0.50000E 01

REAL POLES OF THE UNCOMPENSATED TRANSFER FUNCTION

   -0.35000E 01   -0.60000E 01   -0.90000E 01   -0.15000E 02

COMPLEX CONJUGATES POLES OF UNCOMPENSATED TRANSFER FUNCTION

   -0.15000E 01    0.30000E 01
```

```
OUTPUT DATA
****** ****

FILTER OUTSIDE INTERNAL LOOP, TRANSFER GAIN SPECIFIED

EXTERNAL GAIN.............K=     0.74439E 00
TACHOMETER GAIN..........KT=     0.78757E 01
COMPENSATOR POLE.........PC=     0.11648E 00
COMPENSATOR ZERO.........ZC=     0.78757E 01

COMPUTED ERROR COEFFICIENT IS =     0.50000E 01

COEFFICIENTS OF THE C.E. IN ASCENDING ORDER
    0.19136E 07    0.63472E 06    0.15374E 06

    0.45667E 05    0.82727E 04    0.86074E 03

    0.61387E 02    0.33295E 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY

   -0.51883E 01    0.0

    0.11760E 01   -0.40260E 01

    0.11760E 01    0.40260E 01

   -0.64411E 01   -0.31541E 01

   -0.64411E 01    0.31541E 01

   -0.13594E 01   -0.10980E 02

   -0.13594E 01    0.10980E 02
```

Example 3-3

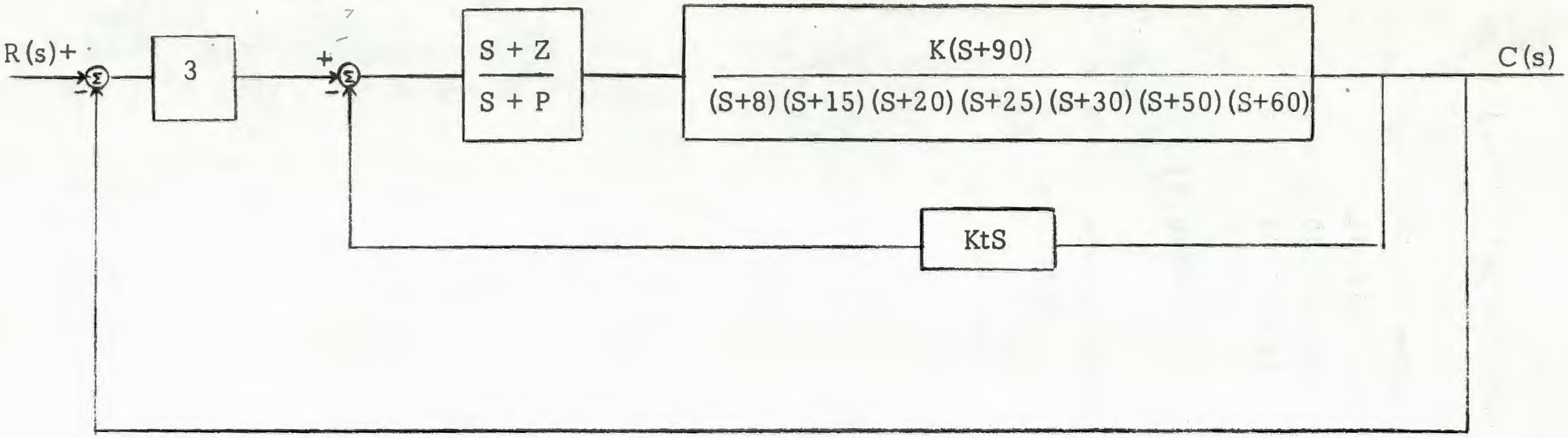


Figure 3-4
Block Diagram for Example 3-3

Design a filter compensator and find values for forward and feedback gains which will give the system shown in Figure 3-4 a constant error coefficient of 0.5 and a pair of complex roots located at $\zeta = 0.5$ and $W_n = 20.0$. It is also specified that a real root be located at $\sigma_x = -100.0$.

```
INPUT DATA
----------


SPECIFICATION FOR ZETA IS........................   0.50000E 00

SPECIFICATION FOR NATURAL FREQUENCY IS...........   0.20000E 02

SPECIFICATION FOR REAL ROOT IS..................   -0.10000E 03

SPECIFICATION FOR ERROR COEFFICIENT IS...........   0.50000E 00

SPECIFICATION FOR EXTERNAL LOOP GAIN IS..........   0.30000E 01

COEFFICIENTS OF NUMERATOR POLYNOMIAL IN DESCENDING ORDER
    0.10000E 01    0.90000E 02

COEFFICIENTS OF DENOMINATOR POLYNOMIAL IN DESCENDING ORDER
    0.10000E 01    0.20800E 03    0.17475E 05    0.76700E 06
    0.18972E 08    0.26382E 09    0.18990E 10    0.54000E 10

REAL ZEROS OF UNCOMPENSATED TRANSFER FUNCTION


   -0.90000E 02

COMPLEX CONJUGATES ZEROS OF UNCOMPENSATED TRANSFER FUNCTION


    0.0            0.0

REAL POLES OF THE UNCOMPENSATED TRANSFER FUNCTION


   -0.80000E 01   -0.15000E 02   -0.20000E 02   -0.25000E 02

   -0.30000E 02   -0.50000E 02   -0.60000E 02

COMPLEX CONJUGATES POLES OF UNCOMPENSATED TRANSFER FUNCTION


    0.0            0.0
```

```
OUTPUT DATA
****** ****

FILTER INSIDE  INTERNAL LOOP, EXTERNAL GAIN SPECIFIED

CHOICE ONE

FORWARD GAIN...............K=     0.12230E 09
TACHOMETER GAIN...........KT=     0.26480E 00
COMPENSATOR POLE..........PC=     0.99600E 02
COMPENSATOR ZERO..........ZC=     0.81438E 01

COMPUTED ERROR COEFFICIENT IS =      0.50000E 00

COEFFICIENTS OF THE C.E. IN ASCENDING ORDER
    0.80676E 12     0.25429E 12     0.31721E 11

    0.21859E 10     0.95365E 08     0.25075E 07

    0.38192E 05     0.30760E 03     0.10000E 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY

   -0.80843E 01    0.0

   -0.11758E 02    0.0

   -0.10000E 02    0.17320E 02

   -0.10000E 02   -0.17320E 02

   -0.71132E 02    0.0

   -0.48312E 02    0.25473E 02

   -0.48312E 02   -0.25473E 02

   -0.99999E 02    0.0
```

```
CHOICE TWO

FORWARD GAIN..............K=     0.87914E 08
TACHOMETER GAIN...........KT=    0.36838E 00
COMPENSATOR POLE..........PC=    0.99600E 02
COMPENSATOR ZERO..........ZC=    0.11329E 02

COMPUTED ERROR COEFFICIENT IS =     0.50000E 00

COEFFICIENTS OF THE C.E. IN ASCENDING ORDER
    0.80676E 12     0.25429E 12     0.31721E 11

    0.21859E 10     0.95365E 08     0.25075E 07

    0.38192E 05     0.30760E 03     0.10000E 01

THE COMPENSATED SYSTEM ROOTS ARE


     REAL PART       IMAGINARY


   -0.80843E 01     0.0


   -0.11758E 02     0.0


   -0.10000E 02     0.17320E 02


   -0.10000E 02    -0.17320E 02


   -0.71132E 02     0.0


   -0.48312E 02     0.25473E 02


   -0.48312E 02    -0.25473E 02


   -0.99999E 02     0.0
```

Example 3-4

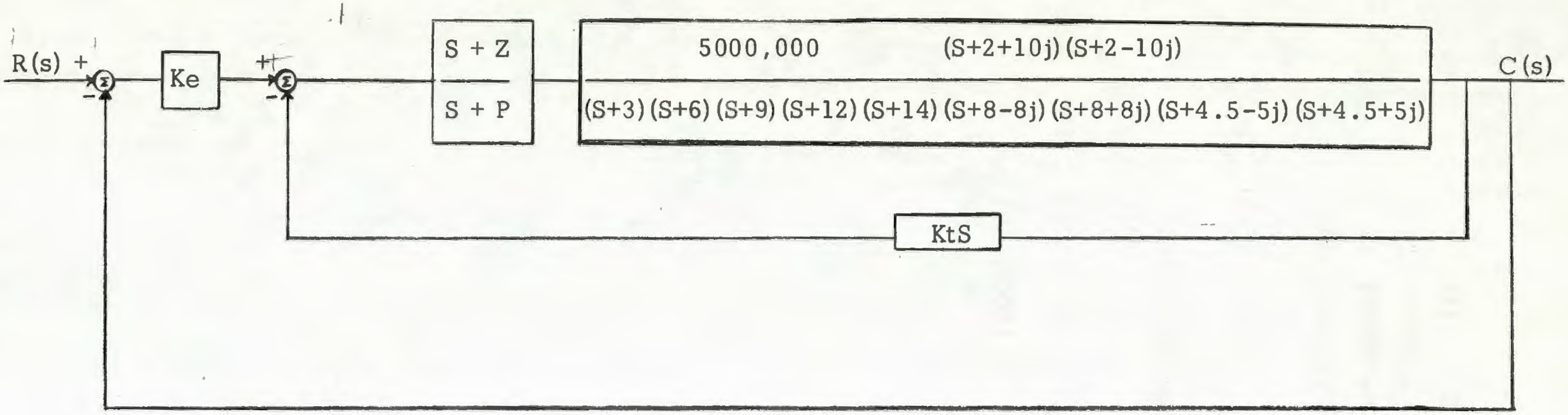

R(s)
+
−
Ke
+
−
S + Z
S + P
5000,000
(S+2+10j) (S+2−10j)
(S+3) (S+6) (S+9) (S+12) (S+14) (S+8−8j) (S+8+8j) (S+4.5−5j) (S+4.5+5j)
C(s)
KtS

Figure 3-5
Block Diagram for Example 3-4

Design a filter compensator and find values for external and feedback gain which will give the system shown in Figure 3-5 a constant error coefficient of 0.2 and a pair of complex roots located at $\zeta = 0.5$ and $W_n = 4.0$. It is also specified that a real root be located at $\sigma_x = -15.0$.

```
INPUT DATA
----------


SPECIFICATION FOR ZETA IS........................   0.50000E 00

SPECIFICATION FOR NATURAL FREQUENCY IS...........   0.40000E 01

SPECIFICATION FOR REAL ROOT IS...................  -0.15000E 02

SPECIFICATION FOR ERROR COEFFICIENT IS...........   0.20000E 00

SPECIFICATION FOR TRANSFER GAIN IS...............   0.50000E 06

COEFFICIENTS OF NUMERATOR POLYNOMIAL IN DESCENDING ORDER
   0.10000E 01    0.40000E 01    0.10400E 03

COEFFICIENTS OF DENOMINATOR POLYNOMIAL IN DESCENDING ORDER
   0.10000E 01    0.69000E 02    0.21522E 04    0.39970E 05
   0.48636E 06    0.40094E 07    0.22356E 08    0.81100E 08
   0.17179E 09    0.15764E 09

REAL ZEROS OF UNCOMPENSATED TRANSFER FUNCTION


   0.0

COMPLEX CONJUGATES ZEROS OF UNCOMPENSATED TRANSFER FUNCTION


  -0.20000E 01    0.10000E 02

REAL POLES OF THE UNCOMPENSATED TRANSFER FUNCTION


  -0.30000E 01   -0.60000E 01   -0.90000E 01   -0.12000E 02

  -0.14000E 02

COMPLEX CONJUGATES POLES OF UNCOMPENSATED TRANSFER FUNCTION


  -0.80000E 01    0.80000E 01   -0.45000E 01    0.50000E 01
```

```
OUTPUT DATA
****** ****

FILTER INSIDE  INTERNAL LOOP, TRANSFER GAIN SPECIFIED

CHCICE CNE

EXTERNAL GAIN.............K=    0.18808E 01
TACHOMETER GAIN..........KT=    0.11514E 00
CCMPENSATOR POLE.........PC=    0.58995E 01
COMPENSATOR ZERO.........ZC=    0.19018E 01

CCMPUTEC ERROR COEFFICIENT IS =    0.20000E 00

COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

     0.11160E 10     0.12874E 10     0.66220E 09

     0.21427E 09     0.46067E 08     0.68786E 07

     0.72216E 06     0.52667E 05     0.25593E 04

     0.74899E 02     0.10000E 01

THE CCMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY        REAL PART        IMAGINARY

   -0.22132E 01     0.0            -0.15003E 02     0.0

   -0.13493E 02    -0.38669E 01    -0.13493E 02     0.38669E 01

   -0.20000E 01    -0.34641E 01    -0.20000E 01     0.34641E 01

   -0.47320E 01    -0.69353E 01    -0.47320E 01     0.69353E 01

   -0.86161E 01    -0.87766E 01    -0.86161E 01     0.87766E 01
```

```
CHOICE TWO

EXTERNAL GAIN.............K=     0.21897E 00
TACHOMETER GAIN..........KT=     0.11514E 00
COMPENSATOR POLE.........PC=     0.58995E 01
COMPENSATOR ZERO.........ZC=     0.16335E 02

COMPUTED ERROR COEFFICIENT IS =     0.20000E 00

COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.11160E 10    0.12874E 10    0.66220E 09

    0.21427E 09    0.46067E 08    0.68786E 07

    0.72216E 06    0.52667E 05    0.25593E 04

    0.74899E 02    0.10000E 01
THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY       REAL PART       IMAGINARY

   -0.22132E 01    0.0            -0.15003E 02    0.0

   -0.13493E 02   -0.38669E 01    -0.13493E 02    0.38669E 01

   -0.20000E 01   -0.34641E 01    -0.20000E 01    0.34641E 01

   -0.47320E 01   -0.69353E 01    -0.47320E 01    0.69353E 01

   -0.86161E 01   -0.87766E 01    -0.86161E 01    0.87766E 01
```

B. ROOTS AND SENSITIVITY SPECIFICATIONS

The problem of design, as was stated in section A, requires as final step the selection by the designer of the "best" system if more than one satisfies the specifications. The "best" system cannot be defined by a simple statement. A given set of specifications can give one or more solutions, and if all of them are physically possible the selection of the "best" system probably will be made taking into account cost, availability of materials, etc.

It is possible for the designer to choose a system in which the dominant root, for example, satisfies the required time response but small variations of a system parameter change its location to positions in which it is not dominant anymore or makes the system unstable. Therefore, it seems obvious that a knowledge of these variations will help the designer choose the "best" system.

One of the many definitions of sensitivity is [4]: The sensitivity of P with respect to Q is the percentage change in P divided by that percentage change in Q which causes the change in P, with all changes considered differentially small.

$$S_Q^P = \frac{d(\ln P)}{d(\ln Q)} = \frac{dP/P}{dQ/Q} = \frac{Q}{P}(dP/dQ) \qquad (3\text{-}13)$$

Kokotovic and Siljak [5] developed algebraic equations to apply the concept of sensitivity as a measure of the quality of the system. They used the microscopic definition of equation 3-13 and the generalized Mitrovic equations, equations 2-3 and 2-4, to develop algebraic sensitivity equations of a system dominant roots with respect to a system parameter. If the parameter considered is the system gain, Kf, the equations can be written as follows:

$$S_{Kf}^{\zeta} = - \frac{K_f D_{Kf}}{\zeta D} \tag{3-14}$$

$$S_{Kf}^{Wn} = -K_f \frac{D_{Kf}^{W}}{D} \tag{3-15}$$

where

$$D = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} \qquad D_{Kf}^{\zeta} = \begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix} \qquad D_{Kf}^{Wn} = \begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}$$

$$A_1 = \sum_{k=o}^{n} (-1)^k K A_k W_n{}^k U_{k-1}(\zeta) \qquad . \qquad A_2 = \sum_{k=o}^{n} (-1)^k K A_k W_n{}^k U_k(\zeta)$$

$$B_1 = \sum_{k=o}^{n} (-1)^k A_k W_n{}^k U'_{k-1}(\zeta) \qquad B_2 = \sum_{k=o}^{n} (-1)^k A_k W_n{}^k U'_k(\zeta)$$

$$C_1 = \sum_{k=o}^{n} (-1)^k \frac{\partial A_k}{\partial K_f} W_n{}^k U_{k-1}(\zeta) \qquad C_2 = \sum_{k=o}^{n} (-1)^k \frac{\partial A_k}{\partial K_f} W_n^k U_k(\zeta)$$

and $U'_k(\zeta)$, $U'_k(\zeta)$ are the derivatives of $U_k(\zeta)$ and $U_{k-1}(\zeta)$ and are related by the equation

$$U'_k(\zeta) = -2\left(\zeta \frac{k-1}{k-2} U'_{k-1}(\zeta) + \frac{k}{K-2} U'_{k-2}(\zeta)\right)$$

Equations 3-14 and 3-15 constitute the required equations to be used in a design problem in which root sensitivity is a specification. They have to be used in conjunction with root specification, that is, root sensitivity specification is not independent of roots specification.

1. General Statement of the Problem

Design the adequate compensator devices required to give (for a specific transfer function) a closed loop time response determined by root locations at $\zeta = \zeta_o$., $W_n = W_o$. It is further required that changes in system gain by X percentage should not change ζ by more than Y percentage.

The statement of the problem implies three specifications, namely damping ratio, natural frequency and sensitivity. Therefore, considering the system gain as a parameter, compensator devices with two unknown parameters must be inserted into the system in order to satisfy the specifications. Then the application of equations 2-3, 2-4 and 3-14 give a set of three nonlinear algebraic equations in three unknowns, as required for solution. Several combinations of compensator devices in cascade and/or feedback may be used. Typical combinations are acceleration and tachometer feedback, or one or more identical sections of filter in cascade. If acceleration and tachometer feedback are used the characteristic equation of an n order uncompensated system (assuming $n \geq 2$) has the following form:

$$F(s) = \sum_{k=o}^{n} G_k(K_f, K_t, K_a) S^k = 0 \tag{3-16}$$

Replacing 3-16 in 2-3, 2-4 and 3-14 with

$$S_{Kf}^{\zeta} = \frac{Y\%}{X\%} = \text{Constant} = S_e$$

the following equations can be written

$$S_e = - \frac{K_f D_{Kf}}{\zeta . D} \tag{3-17}$$

$$\sum_{k=o}^{n} (-1)^k G_k (K_f, K_t, K_a) W_o^k U_{k-1}(\zeta) = 0 \tag{3-18}$$

$$\sum_{k=o}^{n} (-1)^k G_k (K_f, K_t K_a) W_o^k U_k(\zeta) = 0 \tag{3-19}$$

Analytical solution of equations 3-17, 3-18 and 3-19 by hand is, if not impossible (it can be done for very simple systems) at least difficult and time consuming, but if a digital computer is available the numerical solution is fast and accurate.

A digital computer program capable of handling this problem is presented by the author in Appendix D.

Example 3.5

Determine the forward, acceleration and tachometer gains of the system shown in Figure 3-6 such that the closed loop system has a pair of poles at $\zeta = 0.5$ and $W_o = 5.0$. It is also required that if variations of gain are 10% the corresponding of ζ being 5%.

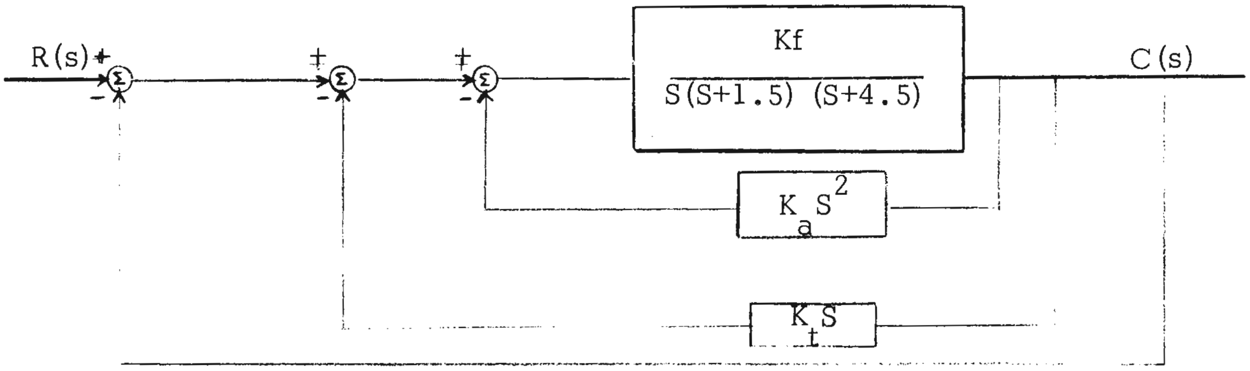


Figure 3.6
Block Diagram for Examples 3.5 and 3.6

In order to be able to use the computer program some preliminary calculations are required.

The open loop transfer function can be written

$$G_o(s) = \frac{G(s)}{1 + G(s)\ (K_a S^2 + K_t S} = \frac{K_f}{S^3 + (6+K_f K_a)S^2 + (6.75+K_f K_t)S}$$

The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G_o(s)}{1 + G_o(s)} = \frac{K_f}{S^3 + (6+K_f K_a)S^2 + (6.75+K_f K_t)S + K_f}$$

Therefore the characteristic equation can be written as

$$CE(s) = A(4)S^3 + A(3)S^2 + A(2)S + A(1)$$

where

$A(4) = 1.0$

$A(3) = 6.0 + K_f K_a = 6.0 + XZ$

$A(2) = 6.75 + K_f K_t = 6.75 + XY$

$A(1) = K_f = X$

taking the partial derivatives of the above coefficients w.r.t. $X(K_f)$

$AD(4) = 0.0$

$AD(3) = Z$

$AD(2) = Y$

$AD(1) = 1.0$

These are the basic calculations required as input to the program in Appendix D, however, the details concerning its use are explained in the program itself.

Two solutions to this problem are shown in the following pages. In both solutions the specifications are exactly satisfied. However, in the first one the real root that the designer does not want to be dominant is located at $\sigma_x = -4.50$ while in the second one that root is located at $\sigma_x = -21.25$. Therefore, it is obvious that between these solutions (the system may have other solutions) solution #2 should be better. It is worthwhile to note that the initial values for the unknown parameters are quite different from the actual ones. However, a good guess implies less iterations and therefore less time of calculation.

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
    0.60000D C3    0.24000D 00    0.40000D-01
    0.30000D CC

SPECIFICATION FOR BANDWIDTH IS              NONE

SPECIFICATION FOR ERROR COEFFICIENT IS      NONE

SPECIFICATION FOR ROOT SENSITIVITY IS       0.20000D 00

SPECIFICATION FOR DAMPING RATIO IS          0.50000D 00

SPECIFICATION FOR NATURAL FREQUENCY IS      0.50000D 01

OUTPUT DATA
****** ****

I.UNKNOWN PARAMETERS
--------- ----------

FORWARD GAIN IS                 0.53125D 03

TACHOMETER GAIN IS              0.23435D 00

ACCELERATION GAIN IS            0.38118D-01

NUMBER OF ITERATIONS IS                  65
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


SENSITIVITY IS                 0.20000D 00

ERROR COEFFICIENT IS           0.40476D 01

III.STABILITY INFORMATION
------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.53125D 03    0.13125D 03    0.26250D 02

    0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY

   -0.25000D 01   -0.43301D 01

   -0.25000D 01    0.43301D 01

   -0.21250D 02    0.0
```

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
    0.10000D 03     0.30000D 00     0.30000D-01
    0.20000D 00

SPECIFICATION FOR BANDWIDTH IS              NONE

SPECIFICATION FOR ERROR COEFFICIENT IS      NONE

SPECIFICATION FOR ROOT SENSITIVITY IS       0.20000D 00

SPECIFICATION FOR DAMPING RATIO IS          0.50000D 00

SPECIFICATION FOR NATURAL FREQUENCY IS      0.50000D 01

OUTPUT DATA
****** ****


I.UNKNOWN PARAMETERS
--------- ----------


FORWARD GAIN IS                 0.11250D 03

TACHOMETER GAIN IS              0.36222D 00

ACCELERATION GAIN IS            0.31111D-01

NUMBER OF ITERATIONS IS                  13
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


SENSITIVITY IS                  0.20000D 00

ERROR COEFFICIENT IS            0.23684D 01

III.STABILITY INFORMATION
-------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.11250D 03    0.47500D 02    0.95000D 01

    0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART      IMAGINARY

   -0.45000D 01   0.0

   -0.25000D 01  -0.43301D 01

   -0.25000D 01   0.43301D 01
```

Example 3.6

Redesign the system of Figure 3-6. Use in this case a root sensitivity constant such that variations in gain of 10% produces variation in ζ. of only 1% (Se = 0.1).

As in example 3.5, two solutions are presented. Comparing these solutions with those of example 3.5 it should be noted that as the system gain is increased the sensitivity constant is reduced and the required roots are more dominant (solution two of example 3.6). This was also verified working the problem with sensitivity specification Se = 0.3 in whose case the resultant parameters for two solutions were:

Solution One	Solution Two
$K = 133.59$	$K = 337.24$
$K_t = 0.3366$	$K_t = 0.2541$
$K_a = 0.325$	$K_a = 0.0370$
$\sigma_x = -5.343$	$\sigma_x = -13.49$

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
     0.80000D 03     0.20000D 00     0.30000D-01
     0.30000D 00

SPECIFICATION FOR BANDWIDTH IS                 NONE


SPECIFICATION FOR ERROR COEFFICIENT IS         NONE


SPECIFICATION FOR ROOT SENSITIVITY IS          0.10000D 00


SPECIFICATION FOR DAMPING RATIO IS             0.50000D 00


SPECIFICATION FOR NATURAL FREQUENCY IS         0.50000D 01


OUTPUT DATA
****** ****


I.UNKNOWN PARAMETERS
--------- ----------


FORWARD GAIN IS                   0.97571D 02


TACHOMETER GAIN IS                0.38704D 00


ACCELERATION GAIN IS              0.29751D-01


NUMBER OF ITERATIONS IS                    32
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


SENSITIVITY IS                  0.10000D 00

ERROR COEFFICIENT IS            0.21919D 01

III.STABILITY INFORMATION
------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

      0.97571D 02     0.44514D 02     0.89028D 01

      0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

        REAL PART       IMAGINARY

     -0.39028D 01    0.0

     -0.25000D 01   -0.43301D 01

     -0.25000D 01    0.43301D 01
```

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
      0.60000D 03     0.24000D 00     0.40000D-01
      0.30000D 00

SPECIFICATION FOR BANDWIDTH IS               NONE


SPECIFICATION FOR ERROR COEFFICIENT IS       NONE


SPECIFICATION FOR ROOT SENSITIVITY IS        0.10000D 00


SPECIFICATION FOR DAMPING RATIO IS           0.50000D 00


SPECIFICATION FOR NATURAL FREQUENCY IS       0.50000D 01


OUTPUT DATA
****** ****


I.UNKNOWN PARAMETERS
--------- ----------



FORWARD GAIN IS                   0.10649D 04


TACHOMETER GAIN IS                0.21714D 00


ACCELERATION GAIN IS              0.39061D-01


NUMBER OF ITERATIONS IS                    25
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


SENSITIVITY IS                0.10000D 00

ERROR COEFFICIENT IS          0.44748D 01

III.STABILITY INFORMATION
------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

     0.10649D 04     0.23799D 03     0.47597D 02

     0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

       REAL PART        IMAGINARY

    -0.25000D 01    -0.43301D 01

    -0.25000D 01     0.43301D 01

    -0.42597D 02     0.0
```

IV. FOUR SPECIFICATIONS DESIGN

The frequency response specification which is most often specified is the bandwidth. It can be defined as the angular frequency, W_b, at which the magnitude of the output to input (closed loop transfer function) has suffered an attenuation M (often expressed in decibels). If the attenuation is three decibels (.707) then the bandwidth is usually called the "3 db. bandwidth." (In this paper this will be the bandwidth to be used.)

A. ROOTS SENSITIVITY AND BANDWIDTH SPECIFICATIONS

In Section III the significance of the root specifications as a measure of the time response of the system and the sensitivity as a measure of the quality of the system were stated. If bandwidth is now added as a specification then it can be said [4] that it is significant because it indicates rise time or speed of response, it measures in part the ability of the system to reproduce the input signal, and it approximately describes the filtering characteristics of the system.

1. Statement of the Problem

Design appropriate compensator devices to give (for a specific transfer function) root locations at $\zeta = \zeta_0$, $W_n = W_o$. It is also required that the compensated system have a bandwidth $W_b = B$ and that variations in gain of X percentage produce variation in ζ_0 of only Y percentage.

The problem as stated here is the same as Section III with the additional constraint of bandwidth. That is, in addition to equations 2-3, 2-4 and 3-17, a fourth equation must be found.

From the definition of 3 db bandwidth the following relationship can be written:

$$\left|\frac{C(jW_b)}{R(jW_b)}\right| = M = (1/2)^{1/2} \tag{4-1}$$

If the system is to have a bandwidth $W_b = B$ then equation 4-1 can be written as

$$\left|\frac{C(jB)}{R(jB)}\right| = 0.707 \tag{4-2}$$

If both sides of equation 4-2 are squared then it can be expressed as a polynomial in B whose coefficients are functions of the unknown parameters, that is, if a closed loop transfer function is assumed to be of the form

$$\frac{C(s)}{R(s)} = \frac{Q_1 S^m + Q_2 S^{m-1} + \ldots\ldots\ldots\ldots + Q_{m+1}}{A_1 S^n + A^2 S^{n-1} + \ldots\ldots\ldots\ldots + A_n +1}$$

then

$$\frac{C(jB)}{R(jB)} = \frac{Q_1 (jB)^m + Q_2 (jB)^{m-1} + \ldots\ldots + Q_{m+1}}{A_1 (jB)^n + A_2 (jB)^{n-1} + \ldots\ldots + A_{n+1}}$$

assuming m and n to be even

$$\left|\frac{C(jB)}{R(jB)}\right|^2 = \frac{(\pm Q_1 B^m \pm Q_3 B^{m-2} \pm \ldots\ldots + Q_{m+1})^2 + (\pm Q_2 B^{m-1} \pm \ldots\ldots + Q_m B)^2}{(\pm A_1 B^n \pm A_3 B^{n-2} \pm \ldots\ldots + A_{n+1})^2 + (\pm A_2 B^{n-1} \pm \ldots\ldots + A_n B)^2} = 1/2$$

or

$$0 = (\underline{+}A_1 B^n \underline{+} \ldots + A_{n+1})^2 + (\underline{+}A_2 B^{n-1} \underline{+} \ldots + A_n B)^2$$
$$- \ 2[(\underline{+}Q_1 B^m \underline{+} \ldots Q_{m+1})^2 + (\underline{+}Q_2 B^{m-1} + \ldots \underline{+}Q_m B)^2] \qquad (4\text{-}3)$$

Equation 4-3 is a rather complicated nonlinear algebraic equation which must be combined with equations 2-3, 2-4 and 3-17 to give the required set of four equations. Compensator devices must now be added to the system in order to satisfy the specifications, that is they altogether must introduce three parameters to combine with the system gain and form a four parameters design problem.

Example 4.1

Design a compensator filter and find the values for forward and tachometer gains to give the system shown in Figure 4-1 a 3 db bandwidth at $W_b = 6.0$ radians/second, a pair of complex poles at $\zeta = 0.5$, $W_n = 5.0$ and a root sensitivity constant of 0.2

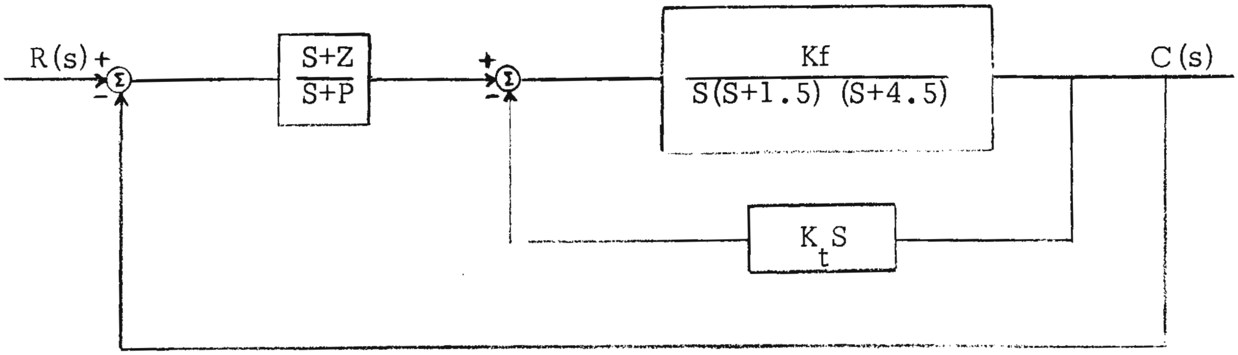


Figure 4-1

Block Diagram for Example 4.1

Solution of this problem was obtained using the computer program of Appendix D. As in Section III the characteristic equation must be found:

The open loop transfer function is

$$G_o(s) = G_e(s)G_c(s) = \frac{K_f(S+Z)}{S(S+P)(S+1.5)(S+4.5) + (S+P)K_fK_tS}$$

where

$$G_e(s) = \frac{K_f}{S(S+1.5)(S+4.5) + K_fK_tS}$$

and

$$G_c(s) = \frac{S + Z}{S + P}$$

the closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K_f(S+Z)}{S(S+P)(S+1.5)(S+4.5) + (S+P)K_fK_tS + K_f(S+Z)}$$

and the characteristic equation can be expressed as

$$CE(s) = A(5)S^4 + A(4)S^3 + A(3)S^2 + A(2)S + A(1)$$

where

$$A(1) = K_fZ$$

$$A(2) = 6.75P + K_fK_tP + K_f$$

$$A(3) = 6.75 + 6P + K_fK_t$$

$$A(4) = 6 + P$$

$$A(5) = 1.0$$

Since root sensitivity has been specified the partial derivatives with respect to the gain K_f are required

$AD(1) = Z$

$AD(2) = K_t P + 1$

$AD(3) = K_a P + K_t$

$AD(4) = K_a$

$AD(5) = 0.0$

With the variables $A(i)$ and $AD(i)$ $(i = 1, \ldots 5)$ and the corresponding specifications as input to the program, a solution is shown in the following pages. This is not a unique solution, it is just one of the many solutions the problem may have. Also, it is worthwhile to observe that the solution doesn't guarantee dominance of the given roots but it does guarantee that a pair of complex roots will be at the specified location and that the other specifications are also met. On the other hand, the fact that information of all the poles of the characteristic equation is obtained makes it possible to investigate by inspection whether or not the given roots are dominant.

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
    0.10000D 03     0.70000D 00     0.10000D 02
    0.30000D 01     0.30000D 00

SPECIFICATICN FCR ERROR COEFFICIENT IS     NONE


SPECIFICATION FOR BANDWIDTH IS             0.60000D 01


SPECIFICATION FCR ROOT SENSITIVITY IS      0.20000D 00


SPECIFICATION FOR DAMPING RATIO IS         0.50000D 00


SPECIFICATION FOR NATURAL FREQUENCY IS     0.50000D 01


OUTPUT DATA
****** ****



I.UNKNOWN PARAMETERS
--------- ----------



FORWARD GAIN IS               0.49027D 02


TACHOMETER GAIN IS            0.91647D 00


COMPENSATOR PCLE IS           0.38918D 01


COMPENSATOR ZERC IS           0.13040D 02


NUMBER OF ITERATIONS IS                34
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


ABS.VALUE OF CLTF AT B IS     0.70711D 00

SENSITIVITY IS                0.20000D 00

ERROR COEFFICIENT IS          0.31786D 01

III.STABILITY INFORMATION
------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.63934D 03     0.25016D 03     0.75033D 02

    0.98918D 01     0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART       IMAGINARY

   -0.25000D 01   -0.43301D 01

   -0.25000D 01    0.43301D 01

   -0.24459D 01   -0.44262D 01

   -0.24459D 01    0.44262D 01
```

B. ROOTS, ERROR COEFFICIENT AND BANDWIDTH SPECIFICATIONS

In earlier sections the basic concepts of bandwidth, error coefficient, sensitivity and root specifications have been discussed. However, since error coefficient was defined in Section III for a particular case the definition can now be generalized and the following equations can be written:

$$K_p = \lim_{s \to 0} G_o(s) \tag{4-4a}$$

$$K_v = \lim_{s \to 0} SG_o(s) \tag{4-4b}$$

$$K_a = \lim_{s \to 0} S^2 G_o(s) \tag{4-4c}$$

where $G_o(s)$ is the open loop transfer function of a negative unity feedback system and the subscripts p, v and a refer to type zero, one or two system.

The error coefficient can be interpreted in terms of the characteristics of system performance in both the time and the frequency domain [4]. From equations 4-4 it is observed that only one of the error constants possesses a finite, non-zero value for any given system and this value describes the low frequency behavior of the open loop transfer function (at low frequencies large error coefficients imply high gains). On the other hand the error coefficients can be interpreted in terms of the steady state errors; i.e., if E(s) is defined as the difference between the Laplace transform of the reference input signal (R(s)) and that of the output signal C(s) in a negative unit feedback system, and if E(s) possesses poles in the left half plane only then the steady state error can be written as

$$E_{ss} = \lim_{s \to o} \frac{SR(s)}{1 + G_o(s)} \qquad (4\text{-}5)$$

From equations 4-4 and 4-5 the following equations can be derived:

$$E_{ss} = \frac{1}{1 + K_p} \qquad \left(\text{for } R(s) = \frac{1}{S}\right) \qquad (4\text{-}6a)$$

$$E_{ss} = \frac{1}{K_v} \qquad \left(\text{for } R(s) = \frac{1}{S^2}\right) \qquad (4\text{-}6b)$$

$$E_{ss} = \frac{1}{K_a} \qquad \left(\text{for } R(s) = \frac{1}{S^3}\right) \qquad (4\text{-}6c)$$

Therefore, K_p, K_v and K_a are measures of the steady state errors if the input is a unit step, a unit ramp and a unit parabolic function respectively.

Example 4.2

Design a filter compensator and find values for forward and feedback gains required to give the system shown in Figure 4-2 a pair of complex poles located at $\zeta = 0.5, W_n = W_o = 5$. It is also required that the system have a bandwidth (3 db bandwidth) at the frequency $W_b = 6$. and a constant error coefficient $K_v = 3.2$.

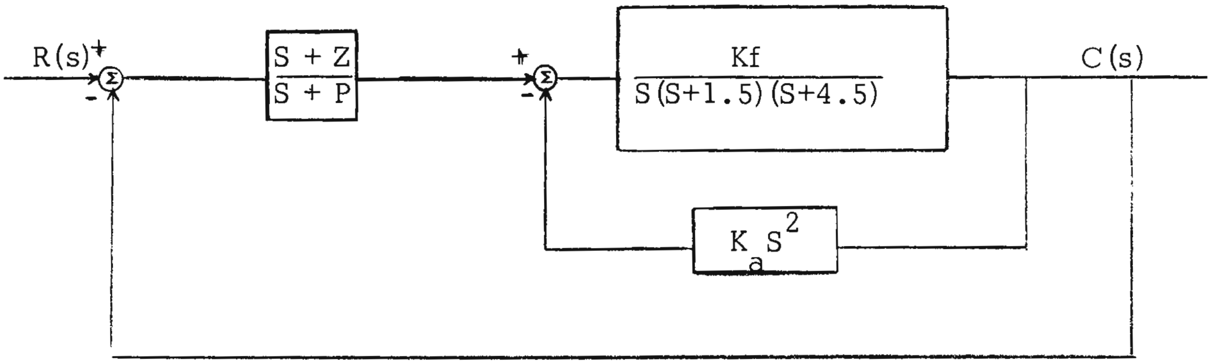


Figure 4-2

Block Diagram of Example 4.2

The characteristic equation of the compensated system is

$$CE(s) = A(5)S^4 + A(4)S^3 + A(3)S^2 + A(2)S + A(1)$$

where

$$A(1) = KZ$$

$$A(2) = 6.75P + K$$

$$A(3) = 6P + 6.75 + KK_aP$$

$$A(4) = 6.0 + KK_aP$$

$$A(5) = 1.0$$

Likewise the bandwidth and error coefficient equations are

$$\left|\frac{C(jB)}{R(jB)}\right| = (1/2)^{1/2} = \frac{[(K_fZ)^2+(K_fB)^2]^{1/2}}{[(A(5)B^4-A(3)B^2+A(1)]^2 + [A(2)B-A(4)B^3)^2]^{1/2}}$$

$$K_v = K_fZ/(A(2) - K_f)$$

The acceleration feedback is probably not the most adequate device to be used in order to meet the specifications. However, the idea of the author was to have a case in which the nonlinearities introduced to the system of equations make difficult the convergence on the method to solve them. This is clearly shown in the solution on the next pages in which the specifications are met but the solution was obtained using a rather large number of iterations (661), see Appendix C, even though the termination criterion was chosen large enough (0.4). On the other hand the results can be considered good from the engineering point of view.

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
    0.40000D 03    0.50000D-02    0.70000D 00
    0.10000D 02    0.40000D 00

SPECIFICATION FOR SENSITIVITY IS           NONE


SPECIFICATION FOR ERROR COEFFICIENT IS    0.32000D 01


SPECIFICATION FOR BANDWIDTH IS            0.60000D 01


SPECIFICATION FOR DAMPING RATIO IS        0.50000D 00


SPECIFICATION FOR NATURAL FREQUENCY IS    0.50000D 01

OUTPUT DATA
****** ****


I.UNKNOWN PARAMETERS
--------- ----------


FORWARD GAIN IS                0.38524D 03


ACCELERATION GAIN IS           0.69901D-02


COMPENSATOR POLE IS            0.12978D 02


COMPENSATOR ZERO IS            0.72768D 00


NUMBER OF ITERATIONS IS                661
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


ABS.VALUE OF CLTF AT B IS    0.70711D 00

SENSITIVITY IS              -0.35906D 00

ERROR COEFFICIENT IS         0.32000D 01

III.STABILITY INFORMATION
------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.28033D 03    0.47285D 03    0.11957D 03

    0.21671D 02    0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART      IMAGINARY

   -0.70219D 00    0.0

   -0.25000D 01   -0.43301D 01

   -0.25000D 01    0.43301D 01

   -0.15969D 02    0.0
```

V. FIVE SPECIFICATIONS DESIGN

In Sections III and IV the problem of design was solved assuming that a maximum of four specifications were required to meet. The problem however can be made much more complex if all the specifications studied so far are to be satisfied at the same time (this is not an unrealistic case but a desired one).

It is easy to see that the system of equations which results when equations 2-3, 2-4, 3-17, 4-3 and 4-4 are combined is a very complicated one even though the system transfer function to be studied is the simplest one. This is due to the fact that since five specifications must be met the number of compensator devices has to be increased in such a way that five unknown parameters are introduced to the system. Under the assumption of variable transfer gain a possible block diagram of a system which is expected to meet five specifications is shown in Figure 5-1 (many combinations can be made but all of them must have a maximum of five unknowns including the system gain).

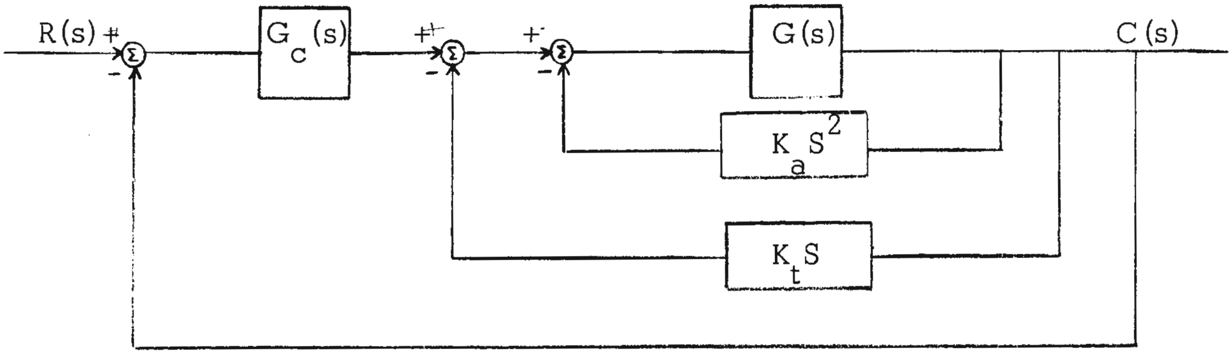


Figure 5-1

Block Diagram for Five Specifications Design

As in the examples studied in the foregoing sections this case can also be handled by the computer program included in Appendix D. The calculations required to use the program can be summarized as follows:

define

$$H(s) = K_a S^2 + K_t S$$

$$G_e(s) = \frac{G(s)}{1 + G(s)H(s)}$$

then the open loop transfer function can be written

$$G_o(s) = G_e(s)G_c(s) \tag{5-1}$$

and the closed loop transfer function as

$$\frac{C(s)}{R(s)} = \frac{G_o(s)}{1 + G_o(s)} \tag{5-2}$$

From equation 5-1 the constant error coefficient is

$$K_x = \lim_{s \to o} S^1 G(s) \tag{5-3}$$

where 1 = 0, 1 or 2 corresponds to x = p, v or a for the three more common types of error coefficients.

From equation 5-2 the characteristic equation can be written as

$$CE(s) = 1 + G_o(s) \tag{5-4}$$

and the 3 db bandwidth equation as

$$\left|\frac{C(jW_b)}{R(jW_b)}\right| = \frac{|G_o(jW_b)|}{|1 + G_o(jW_b)|} = (1/2)^{1/2} \tag{5-5}$$

Equations 5-3, 5-4 and 5-5 are the necessary equations required as input to the computer program.

The fact that five specifications must be met doesn't imply that the system can be forced by using a prescribed set of compensator devices to meet those specifications. Hence the designer must decide if all the specifications cannot be met, which of them should be sacrificed in favor of the others or in favor of the impossibility of using another type of compensator device. With this in mind and taking the block diagram of Figure 5-1 as an example, the flowchart of Figure 5-2 was realized in the computer program of Appendix D. However, it is worthwhile to note that the program was written to handle the characteristic equation of the compensated system of three, four or five parameters and, therefore, the flowchart of Figure 5-2 is just an example that uses the configuration of Figure 5-1.

Example 5.1

Design a filter compensator and find values for forward and feedback gains required to give the system shown in Figure 5-3 a pair of complex poles located at $\zeta = 0.5$, $W_n = 5.0$, a 3 db bandwidth at frequency B = 6.0 radians/second, a constant error coefficient $K_v = 5.0$ and a root sensitivity constant Se = 0.2.

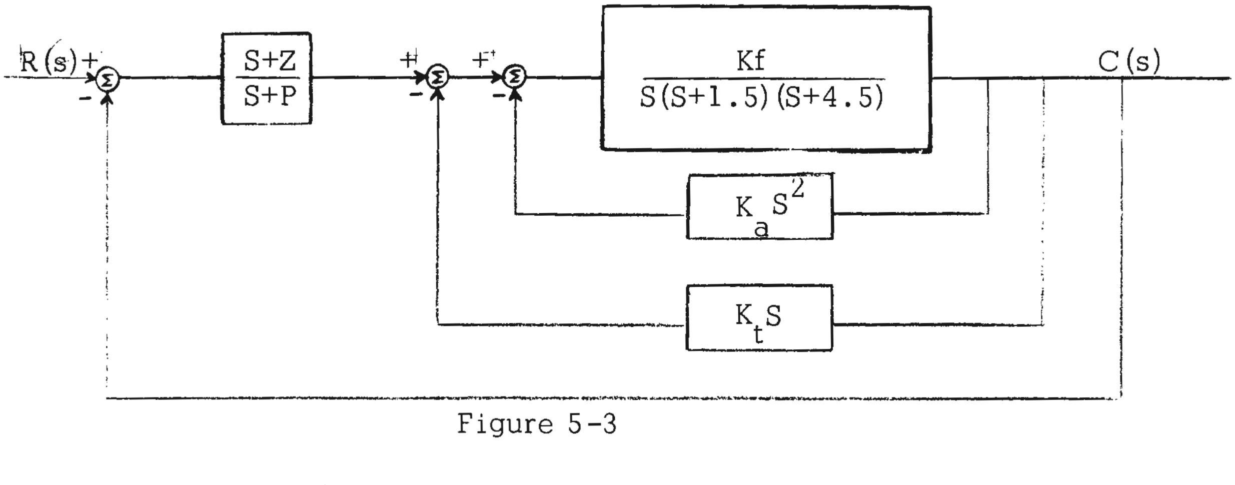


Figure 5-3

Block Diagram for Example 5.1

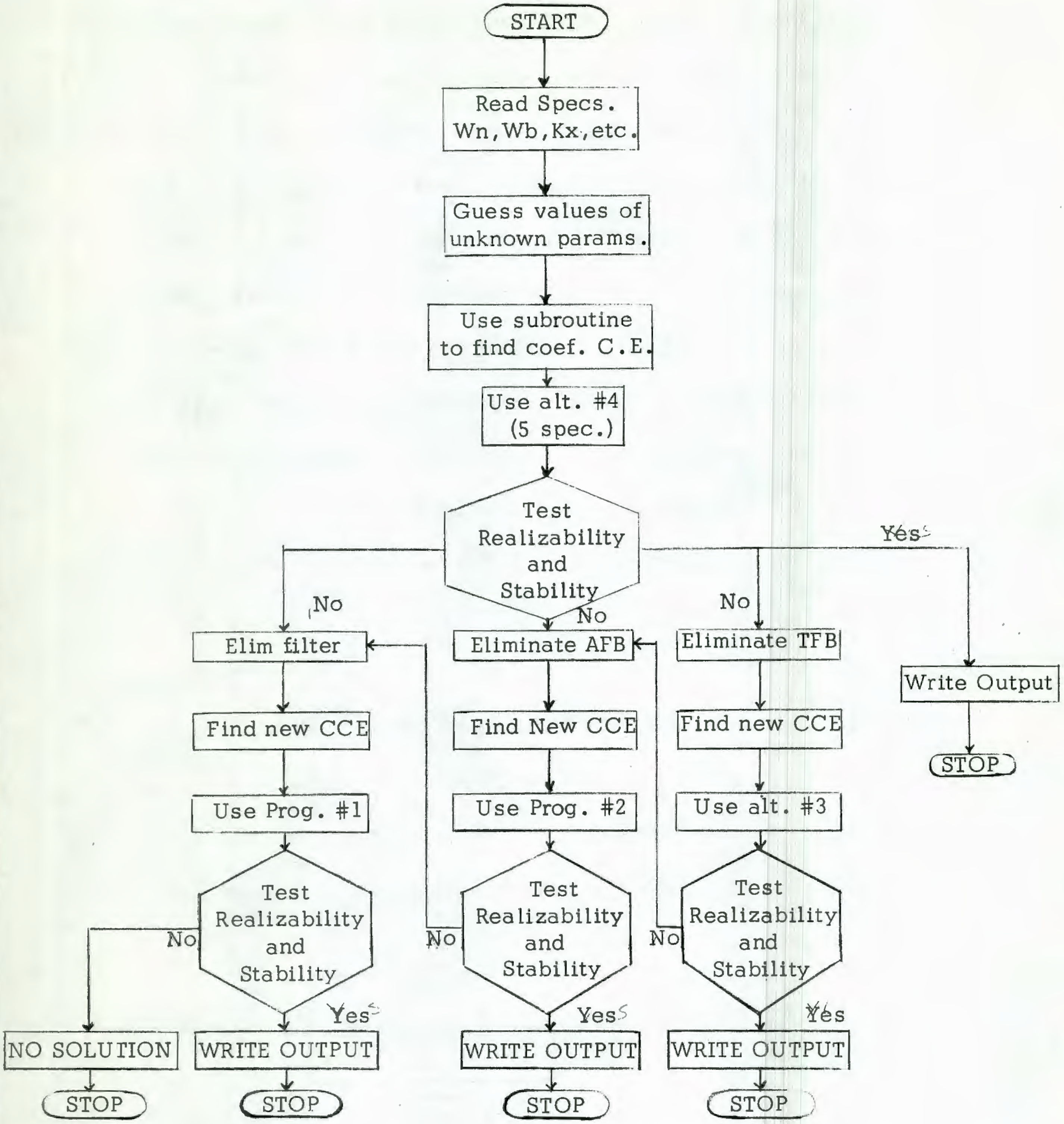


Figure 5-2

Flowchart showing the design procedure (given a set of five specifications) for block diagram of Figure 5-1

Following the steps on page 62 equations 5-3, 5-4 and 5-5 can be written, for this particular case, as follows:

$$K_v = (K_f Z)/(A(2) - K_f)$$

$$\left|\frac{C(jB)}{R(jB)}\right| = (1/2)^{1/2} = \frac{[(AN(1) - AN(3)B^2)^2 + (AN(2)B - AN(4)B^3)^2]^{1/2}}{[(A(1) - A(3)B^2 + A(5)B^4 - A(7)B^6) + (A(2)B - A(4)B^3 + A(6)B^5)^2]^{1/2}}$$

$$CE(s) = A(5)S^4 + A(4)S^3 + A(3)S^2 + A(2)S + A(1)$$

where

$A(1) = K_f Z$

$A(2) = 6.75P + K_f K_t P + K_f$

$A(3) = 6P + K_f K_a P + K_f K_t + 6.75$

$A(4) = K_f K_a P + 6$

$A(5) = 1.0$

the partial derivatives of the coefficients A(i) with respect to the system gain are given by

$AD(1) = Z$

$AD(2) = K_t P + 1.0$

$AD(3) = K_t$

$AD(4) = 0.0$

$AD(5) = 0.0$

With this data a solution was obtained and is shown in the following pages. Comparing this solution with examples 3.5 and 4.1, the following remarks can be written:

a. In all the cases the specifications are exactly met.

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
     0.50000D 02     0.90000D 00     0.13000D 02
     0.39000D 01     0.20000D-01     0.20000D 00


SPECIFICATION FOR ROOT SENSITIVITY IS      0.20000D 00


SPECIFICATION FOR BANDWIDTH IS             0.60000D 01


SPECIFICATION FOR ERROR COEFFICIENT IS     0.32000D 01


SPECIFICATION FOR DAMPING RATIO IS         0.50000D 00


SPECIFICATION FOR NATURAL FREQUENCY IS     0.50000D 01

OUTPUT DATA
****** ****


I.UNKNOWN PARAMETERS
--------- ----------


FORWARD GAIN IS                 0.51132D 02


TACHOMETER GAIN IS              0.87909D 00


ACCELERATION GAIN IS            0.12020D-02


COMPENSATOR POLE IS             0.38503D 01


COMPENSATOR ZERO IS             0.12458D 02


NUMBER OF ITERATIONS IS                  11
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


ABS.VALUE OF CLTF AT B IS    0.70711D 00

SENSITIVITY IS               0.20001D 00

ERROR COEFFICIENT IS         0.32000D 01

III.STABILITY INFORMATION
-------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.63699D 03    0.25019D 03    0.75038D 02

    0.99117D 01    0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART      IMAGINARY

   -0.25000D 01   -0.43301D 01

   -0.25000D 01    0.43301D 01

   -0.24559D 01   -0.44100D 01

   -0.24559D 01    0.44100D 01
```

b. Dominance of the given pair of complex poles is only met, for the same system, with the three specifications problem.

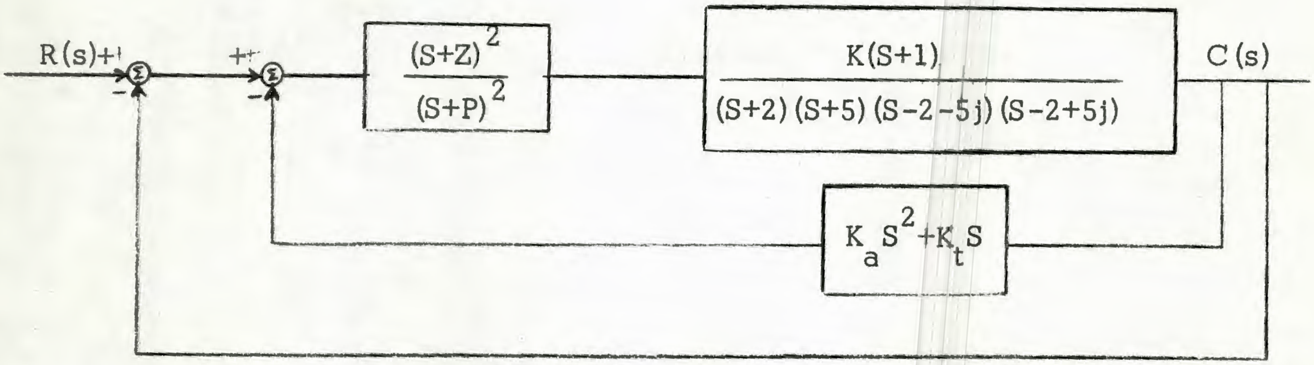
c. System parameters for examples 4.1 and 5.1 are comparable.

d. More than one solution can be obtained in each case (for simplicity in this paper only one solution is shown for examples 4.1 and 5.1).

e. Each of the possible solutions is greatly dependent on the initial guess.

Example 5.2

The system shown in Figure 5-4 is completely unstable due to the presence of a pair of complex poles on the right hand plane. Design two identical sections of filter and find the forward and feedback gains required to give the compensated system a pair of complex poles located at $\zeta = 0.5$, $W_n = 5.0$; a 3 db bandwidth at a frequency $W_b = 1.0$ radians/second; a constant error coefficient $K_p = 2.0$ and a root sensitivity constant Se = 0.2



R(s)
+
−
Σ
+
−
Σ
$\frac{(S+Z)^2}{(S+P)^2}$
$\frac{K(S+1)}{(S+2)(S+5)(S-2-5j)(S-2+5j)}$
C(s)
$K_a S^2 + K_t S$

Figure 5-4

Block Diagram for Example 5.2 and 5.3

Following the same steps as example 5.1 the equations can be written:

$$K_p = (K_f Z^2)/290P^2$$

$$CE(s) = A(7)S^6 + A(6)S^5 + A(5)S^4 + A(4)S^3 + A(3)S^2 + A(2)S + A(1)$$

$$\left|\frac{C(jB)}{R(jB)}\right| = (1/2)^{1/2} = \frac{[(K_f Z)^2 + (K_f B)^2]^{1/2}}{[(A(5)B^4 - A(3)B^2 + A(1))^2 + (A(2)B - A(4)B^3)^2]^{1/2}}$$

where the following definitions were made:

$AN(1) = K_f Z^2$

$AN(2) = 2K_f Z + K_f Z^2$

$AN(3) = 2K_f Z + K_f$

$AN(4) = 1.0$

$A(1) = 290P^2 + K_f Z^2$

$A(2) = 580P + 163P^2 + K_f K_t Z^2 + 2K_f Z + K_f Z^2$

$A(3) = 290+326P+11P^2+K_f K_a Z^2+2K_f K_t Z+K_f K_t Z^2+2K_f Z+K_f$

$A(4) = 163+22P+3P^2+2K_f K_a Z+K_f K_a Z^2+2K_f K_t Z+K_f K_t+K_f$

$A(5) = 11 + 6P + P^2 + 2K_f K_a Z + K_f K_a + K_f K_t$

$A(6) = 1.0$

Taking the partial derivatives with respect to the system gain,

$AD(1) = Z^2$

$AD(2) = K_t Z^2 + 2Z + Z^2$

$AD(3) = K_a Z^2 + 2K_t Z + K_t Z^2 + 2Z + 1.0$

$AD(4) = K_a Z^2 + 2K_a Z + 2K_t Z + K_t + 1.0$

$AD(5) = 2K_a Z + K_a + K_t$

$AD(6) = K_a$

$AD(7) = 0.0$

From the above definitions it is easy to imagine the complexity of the resultant equations when the bandwidth and sensitivity equations are expanded. These are practically impossible to solve without a computer. Using the computer program in Appendix D, a solution was obtained and is shown in the following pages.

The solution of this system indicates that in order to meet the specifications the compensator zero of the double section of filter must be replaced on the right hand plane. By inspection of the compensated system roots the system is stable. This means that starting from a completely unstable system the result after compensation is a stable system satisfying all five specifications. A much more reasonable answer would have been if the compensator zero can also be moved to the left hand plane. This, however, is very difficult to meet with the given combination of compensator devices and required specifications.

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
    0.35000D 02    0.10000D 01    -0.20000D 01
    0.40000D 00    0.10000D 00     0.30000D 00


SPECIFICATION FOR ROOT SENSITIVITY IS      0.20000D 00

SPECIFICATION FOR BANDWIDTH IS             0.10000D 01

SPECIFICATION FOR ERROR COEFFICIENT IS     0.20000D 01

SPECIFICATION FOR DAMPING RATIO IS         0.50000D 00

SPECIFICATION FOR NATURAL FREQUENCY IS     0.50000D 01

OUTPUT DATA
****** ****


I.UNKNOWN PARAMETERS
--------- ----------


FORWARD GAIN IS                0.37526D 02

TACHOMETER GAIN IS             0.10684D 01

ACCELERATION GAIN IS           0.89858D-01

COMPENSATOR POLE IS            0.41001D 00

COMPENSATOR ZERO IS           -0.16119D 01

NUMBER OF ITERATIONS IS                 21
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


ABS.VALUE OF CLTF AT B IS     0.70711D 00

SENSITIVITY IS                0.20000D 00

ERROR COEFFICIENT IS          0.20000D 01

III.STABILITY INFORMATION
------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

     0.14626D 03     0.34591D 03     0.32574D 03

     0.11878D 03     0.46223D 02     0.71921D 01

     0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

      REAL PART       IMAGINARY

   -0.71558D 00    -0.42134D 00

   -0.71558D 00     0.42134D 00

   -0.38045D 00    -0.28878D 01

   -0.38045D 00     0.28878D 01

   -0.25000D 01    -0.43301D 01

   -0.25000D 01     0.43301D 01
```

Example 5.3

Redesign the system of example 5.2 with a 3 db bandwidth at the frequency B = 0.34 and with a root sensitivity constant of $S_e = 0.16$. Inspection of the results shown in the following pages indicates that the given specifications are exactly met and the system is stable with all its poles and zeros on the left hand plane. Also it is worthwhile to observe the increase in system gain as the sensitivity constant was reduced.

```
INPUT DATA
***** ****

INITIAL VALUES FOR UNKNOWN PARAMETERS ARE
     C.90000D C3     0.20000D 00     0.50000D 00
     C.70000D CC     0.50000D-01     0.40000D 0C


SPECIFICATICN FCR ROOT SENSITIVITY IS      0.16000D 00

SPECIFICATION FOR BANDWIDTH IS             0.34000D 00

SPECIFICATION FOR ERROR COEFFICIENT IS     0.20000D 01

SPECIFICATION FCR DAMPING RATIO IS         0.50000D 00

SPECIFICATION FOR NATURAL FREQUENCY IS     0.50000D 01

OUTPUT DATA
****** ****


I.UNKNCWN PARAMETERS
--------- ----------


FORWARD GAIN IS               0.94291D 03

TACHOMETER GAIN IS            0.25084D 00

ACCELERATION GAIN IS          0.47869D-01

COMPENSATOR PCLE IS           0.84389D 00

COMPENSATOR ZERO IS           0.66186D 00

NUMBER OF ITERATIONS IS               379
```

```
II.CALCULATED SPECIFICATIONS
------------- --------------


ABS.VALUE OF CLTF AT B IS     0.70711D 00

SENSITIVITY IS                0.16000D 00

ERROR COEFFICIENT IS          0.20000D 01

III.STABILITY INFORMATION
-------------- -----------


COEFFICIENTS OF THE C.E. IN ASCENDING ORDER

    0.61957D 03    0.23703D 04    0.32005D 04

    0.17557D 04    0.35817D 03    0.49823D 02

    0.10000D 01

THE COMPENSATED SYSTEM ROOTS ARE

     REAL PART      IMAGINARY

   -0.74002D 00   -0.10591D 00

   -0.74002D 00    0.10591D 00

   -0.10485D 01    0.0

   -0.25000D 01   -0.43301D 01

   -0.25000D 01    0.43301D 01

   -0.42295D 02    0.0
```

VI. CONCLUSIONS AND RECOMMENDATIONS

The purpose of this paper was to use a digital computer to design a linear control system, which must satisfy up to five specifications, using algebraic methods. The feasibility of doing this was established and several examples showed that within certain limitations the method is applicable.

Two computer programs were written by the author using in both algebraic equations, developed by other authors in earlier work, to express specifications such as root locations, bandwidth, root sensitivity and error coefficient. The first program is accurate, fast and capable of handling root and error coefficient specifications to design a linear control system whose differential equations are available and can be expressed as a ratio of two polynomials in the complex frequency domain (transfer function). A fixed configuration of compensator devices consisting of a cascaded single filter and velocity (tachometer) feedback was used and the resultant nonlinear algebraic equations were solved by a very straightforward method of linearization. The program was written to handle uncompensated transfer functions up to nine order denominator and seven order numerator, however, implementation for higher order cases is apparent. On the other hand the program was written to give two solutions for a particular system if all the parameters are physically realizable. In addition, it was provided the possibility of getting only one solution or simply no solution when the parameters are not physically realizable.

As a consequence of increasing the number of specifications a different approach was needed for writing the second program. That is, the presence of specifications as root sensitivity and bandwidth made it impossible to use a linearization method to solve the resultant nonlinear equations. Therefore, an interpolative procedure (Appendix C) was required and several subroutines and external functions sub-programs were written in conjunction with a rather general program based on the characteristic equation of the particular system to be designed. The examples showed the accuracy of the design and the dependence of the results on the set of initial values for the unknown parameters guessed by the designer. This set of initial values constitute the main disadvantage of the technique (the method may diverge for poor initial guess for the unknown parameters). One way of implementing the initial guess could be by using some kind of search procedure as it is used in some professional subroutines (this is not shown in this paper). Another way used by the author, can be summarized as follows:

a. Use program ROSDEP (roots and error specification design program) and find the unknown parameters required to meet root and error specifications using single section of filter in cascade and tachometer feedback as compensator devices.

b. From the characteristic equation of the compensated system (actual problem) find the values of root sensitivity and frequency at 3 db bandwidth. This can be done using the same equations required as input for the actual program of design, FISDEP, by a very simple auxiliary program whose flowchart is indicated in Figure 6-1.

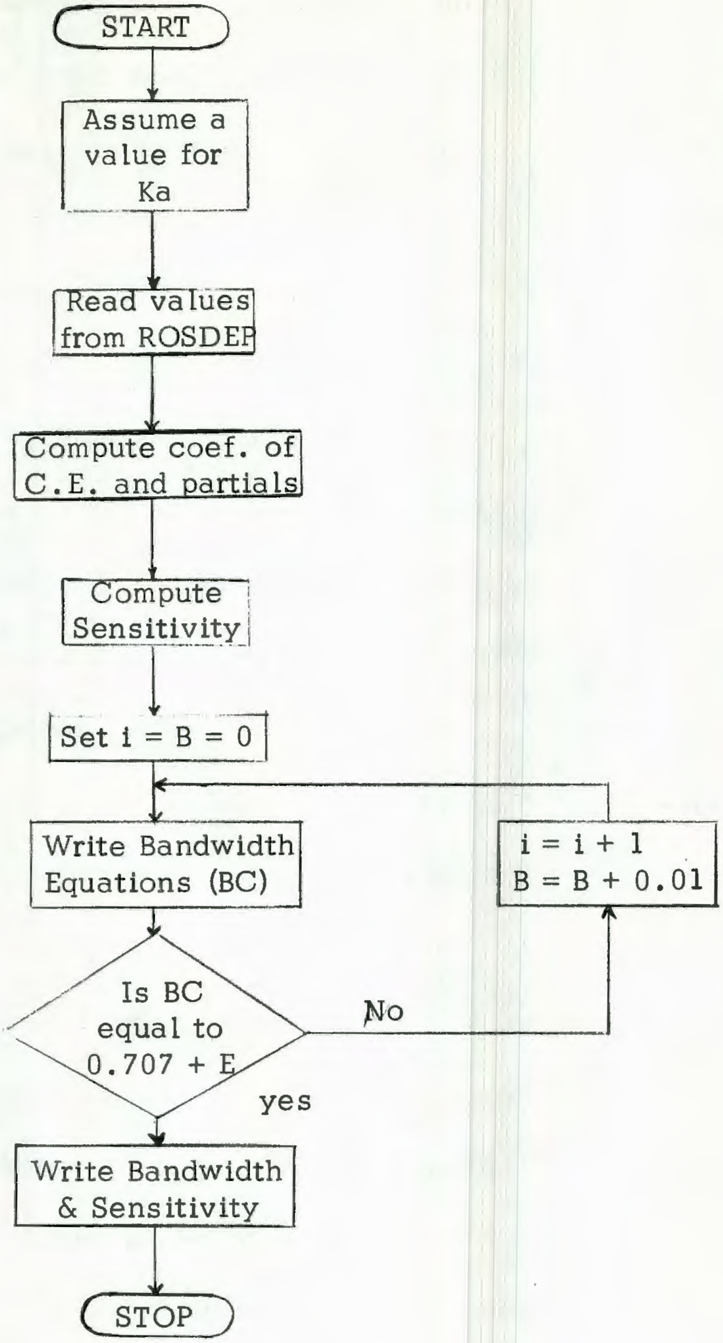


Figure 6-1

Flowchart showing sensitivity and Bandwitch calculations

c. Compare the values obtained for sensitivity and bandwidth with the corresponding specifications and make reasonable adjustments according to the configuration of compensator devices required to use and some rules as:

1. The sign of the sensitivity constant is only a measure of direction.

2. There is a tendency (verified for some systems) to increase the gain as the sensitivity decreases.

The reason for the author to suggest this procedure is based on the fact that during the research he found that for several transfer functions studied the roots and error coefficient specifications were satisfied (even though the initial guesses were taken at random), but the sensitivity and bandwidth specifications were much more difficult to satisfy. Therefore, a knowledge of what these quantities will be leads to a better guess and helps to insure the convergence of the procedure. On the other hand the total computational time using the suggested approach is increased only by about 30 seconds compared with the computational time required for a complete run taking directly the guessed values.

In conclusion it was shown in this paper that a digital computer can be used to design linear control systems and that the method insures neither dominance of the given complex poles nor physical realizability of the system parameters, but it does insure that the specifications will be met and stability information can be obtained from the compensated system roots. Furthermore, the auxiliary program suggested to help in finding the initial guess values to be used in program FISDEP (five specifications design program) provided a trivial method of computing root sensitivity and 3 db bandwidth for a root and error specifications design problem whose solution was obtained using program ROSDEP.

APPENDIX A

TABLE OF CHEBYCHEV's FUNCTIONS ($U_k(\xi)$)

ξ	$U_{-1}(\xi)$	$U_o(\xi)$	$U_1(\xi)$	$U_2(\xi)$	$U_3(\xi)$	$U_4(\xi)$	$U_5(\xi)$	$U_6(\xi)$
0.0	-1.000	0.0	-1.000	-0.0	1.000	-0.0	-1.000	-0.0
0.050	-1.000	0.0	-1.000	0.100	0.990	-0.199	-0.970	0.296
0.100	-1.000	0.0	-1.000	0.200	0.960	-0.392	-0.882	0.568
0.150	-1.000	0.0	-1.000	0.300	0.910	-0.573	-0.738	0.794
0.200	-1.000	0.0	-1.000	0.400	0.840	-0.736	-0.546	0.954
0.250	-1.000	0.0	-1.000	0.500	0.750	-0.875	-0.313	1.031
0.300	-1.000	0.0	-1.000	0.600	0.640	-0.984	-0.050	1.014
0.350	-1.000	0.0	-1.000	0.700	0.510	-1.057	0.230	0.896
0.400	-1.000	0.0	-1.000	0.800	0.360	-1.088	0.510	0.680
0.450	-1.000	0.0	-1.000	0.900	0.190	-1.071	0.774	0.374
0.500	-1.000	0.0	-1.000	1.000	0.000	-1.000	1.000	0.000
0.550	-1.000	0.0	-1.000	1.100	-0.210	-0.869	1.166	-0.413
0.600	-1.000	0.0	-1.000	1.200	-0.440	-0.672	1.246	-0.824
0.650	-1.000	0.0	-1.000	1.300	-0.690	-0.403	1.214	-1.175
0.700	-1.000	0.0	-1.000	1.400	-0.960	-0.056	1.038	-1.398
0.750	-1.000	0.0	-1.000	1.500	-1.250	0.375	0.688	-1.406
0.800	-1.000	0.0	-1.000	1.600	-1.560	0.896	0.126	-1.098
0.850	-1.000	0.0	-1.000	1.700	-1.890	1.513	-0.682	-0.353
0.900	-1.000	0.0	-1.000	1.800	-2.240	2.232	-1.778	0.968
0.950	-1.000	0.0	-1.000	1.900	-2.610	3.059	-3.202	3.025
1.000	-1.000	0.0	-1.000	2.000	-3.000	4.000	-5.000	6.000

APPENDIX B
FIRST DERIVATIVE OF SOME CHEBYCHEV's FUNCTIONS ($U'_k(\xi)$)

ξ	$U'_2(\xi)$	$U'_3(\xi)$	$U'_4(\xi)$	$U'_5(\xi)$	$U'_6(\xi)$	$U'_7(\xi)$	$U'_8(\xi)$
0.0	2.000	0.0	-4.000	-0.0	6.000	0.0	-8.000
0.050	2.000	0.400	-3.940	-1.192	5.761	2.360	-7.406
0.100	2.000	0.800	-3.760	-2.336	5.056	4.484	-5.695
0.150	2.000	1.200	-3.460	-3.384	3.921	6.149	-3.076
0.200	2.000	1.600	-3.040	-4.288	2.416	7.163	0.121
0.250	2.000	2.000	-2.500	-5.000	0.625	7.375	3.469
0.300	2.000	2.400	-1.840	-5.472	-1.344	6.693	6.477
0.350	2.000	2.800	-1.060	-5.656	-3.359	5.097	8.641
0.400	2.000	3.200	-0.160	-5.504	-5.264	2.652	9.494
0.450	2.000	3.600	0.860	-4.968	-6.879	-0.474	8.674
0.500	2.000	4.000	2.000	-4.000	-8.000	-4.000	6.000
0.550	2.000	4.400	3.260	-2.552	-8.399	-7.514	1.556
0.600	2.000	4.800	4.640	-0.576	-7.824	-10.460	-4.212
0.650	2.000	5.200	6.140	1.976	-5.999	-12.125	-10.391
0.700	2.000	5.600	7.760	5.152	-2.624	-11.621	-15.483
0.750	2.000	6.000	9.500	9.000	2.625	-7.875	-17.281
0.800	2.000	6.400	11.360	13.568	10.096	0.389	-12.735
0.850	2.000	6.800	13.340	18.904	20.161	14.663	2.200
0.900	2.000	7.200	15.440	25.056	33.216	36.668	32.715
0.950	2.000	7.600	17.660	32.072	49.681	68.371	85.315
1.000	2.000	8.000	20.000	40.000	70.000	111.999	167.998

APPENDIX C

The algorithm summarized in the flowchart of Figure C-1 was used to write the subroutines to solve the nonlinear equations resultant from a five (or less) specifications design problem. "It is a variation of Newton's method, in which the partial derivatives are replaced by the multidimensional analogues of difference quotients. It produces superlinear convergence (of order $1/2\ (1+\sqrt{5})$ or approximately 1.62) to a simple zero, provided the given functions are twice continuously differentiable in a neighborhood of the zero, and the initial approximations are close enough." [6]

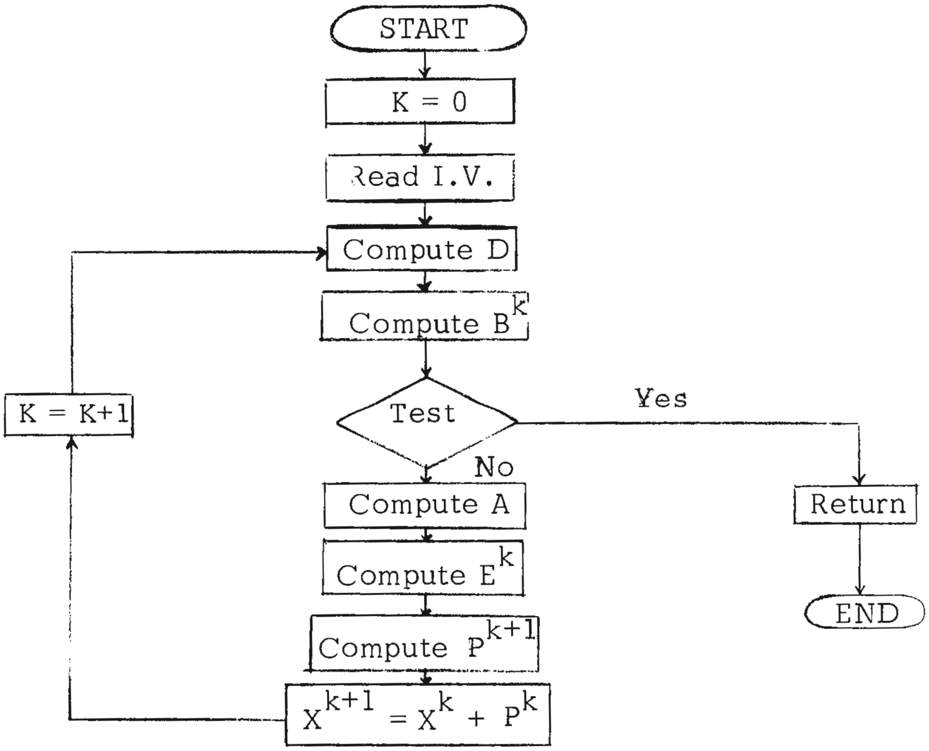


Figure C-1

Flowchart showing the procedure
used to solve a set of nonlinear equations

The capital letters indicated in the flowchart of Figure 3-1 refer to matrices (or vectors). To compute them the following definitions are required:

$$D = \|X^k - X^{k-1}\| \; H^k$$

where

$\|X^k - X^{k-1}\|$ is the Euclidean norm of the vector X

$$H^k = \begin{bmatrix} P_1^k P_2^k & P_1^k P_3^k \ldots\ldots\ldots\ldots P_1^k P_n^k & & -P_1^k \\ -(P_1^k)^2 & P_2^k P_3^k \ldots\ldots\ldots\ldots P_2^k P_n^k & & -P_2^k \\ 0 & -((P_1^k)^2 + (P_2^k)^2) \ldots & & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \\ 0 & 0 \ldots\ldots\ldots\ldots\ldots & -\sum_{j=1}^{n-1} (P_j^k)^2 & -P_n^k \end{bmatrix} C$$

C is a diagonal matrix chosen to normalize the columns to length 1

$$P^{k+1} = X^{k+1} - X^k$$

$$B^k = f(X^k)$$

$$A = f(X^k + D^k) - B^k$$

$$E^k = A^{-1} B^k$$

$$P^{k+1} = D^k E^k$$

Using these definitions and $X = [x \; y \; z \; p \; r]^t$ and $P = [dx \; dy \; dz \; dp \; dr]^t$ for a system of five equations and five unknown the following matrix can be written:

$$D=\begin{bmatrix} sdy/u & sdxdz/vu & sdxdp/wv & dxdr/w & -dx \\ -sdx/u & sdydz/vu & sdydp/wv & dydr/w & -dy \\ 0 & -su/v & sdzdp/wv & dzdr/w & -dz \\ 0 & 0 & -vs/w & dpdr/w & -dp \\ 0 & 0 & 0 & -w & -dr \end{bmatrix}$$

where

$$u = (dx^2 + dy^2)^{1/2}$$
$$v = (dx^2 + dy^2 + dz^2)^{1/2}$$
$$w = (dx^2 + dy^2 + dz^2 + dp^2)^{1/2}$$
$$s = (dx^2 + dy^2 + dz^2 + dp^2 + dr^2)^{1/2}$$

The vector B and matrix A were computed using an external function (f) in which the nonlinear equations were set in terms of the unknown parameters. Likewise the vector E was calculated using an external subroutine for solving linear algebraic equations.

Different subroutines were written for the cases of three, four and five simultaneous equations. In all of them a maximum of one thousand iterations were considered and indication of convergence was provided by using a counter which finally will be output of the main program. If the number of iterations in the output is indicated by a zero the procedure didn't converge and another set of initial values should be tried. Other parameters in the subroutines are self explanatory.

```
C         *******ROOTS AND ERROR COEFFICIENT DESIGN PROGRAM(ROSDEP)*********
C
C
C         THIS PROGRAM USES ROOT SPECIFICATIONS AND ERROR COEFFICIENT TO  DE
C         SIGN A CONTROL SYSTEM WHOSE TRANSFER FUNCTION, G(S), IS   CASCADED
C         WITH A COMPENSATOR DEVICE,GC(S),AND A TACHOMETER FEEDBACK COMPENSA
C         TOR. GC(S) IS DESIGNED TO BE A SINGLE SECTION OF FILTER INSIDE  OR
C         OUTSIDE THE INTERNAL LOOP FORMED BY TACHOMETER AND TRANSFER   FUNC
C         TION.AN EXTERNAL LOOP AROUND COMPENSATORS AND PLANT USES  NEGATIVE
C         UNITY FEEDBACK.G(S) IS ASSUMED AS A TRANSFER FUNCTION   DESCRIBING
C         THE UNCOMPENSATED SYSTEM AS A RATIO OF TWO POLYNOMIALS IN THE S DO
C         MAIN.I.E.THE LAPLACE'S TRANSFORM OF THE SYSTEM DIFFERENTIAL   EQUA
C         TIONS.THIS TRANSFER FUNCTION MAY BE EITHER IN FACTORED FORM OR  AS
C         A RATIO OF POLYNOMIALS OF KNOWN COEFFICIENTS.THE ORDER OF THE NUME
C         RATOR IS ASSUMED TO BE LESS THAN OR EQUAL TO SEVEN AND THAT OF THE
C         DENOMINATOR LESS THAN OR EQUAL TO NINE.
C         THE PROGRAM REQUIRES AS INPUT A SET OF TEN DATA CARDS  ORDERED  IN
C         THE FOLLOWING MANNER:
C
C         DATA CARD # 01 IN FORMAT 5I5 THE ORDER OF THE DENOMINATOR TRANSFER
C                        FUNCTION(N),THE NUMERATOR TRANSFER FUNCTION(M), THE
C                        INTEGER(NT) ZERO OR ONE DEPENDING ON WHETHER OR NOT
C                        THE POLYNOMIALS ARE IN FACTORED FORM(DATA CARDS NI-
C                        NE AND TEN SHOULD BE OMITTED IF NT IS ZERO.LIKEWISE
C                        DATA CARDS THREE THROUGH EIGHT SHOULD BE OMITTED IF
C                        NT IS ONE),THE INTEGER(INSF) ZERO OR ONE  DEPENDING
C                        ON THE POSITION OF THE FILTER,THAT IS:
C                        A. ZERO:IMPLIES FILTER OUTSIDE THE INTERNAL LOOP
C                        B.  ONE:IMPLIES FILTER INSIDE THE INTERNAL LOOP AND
C                        THE INTEGER(ING) ZERO OR ONE DEPENDING RESPECTIVELY
C                        ON WHETHER THE GAIN TRANSFER FUNCTION IS   VARIABLE
C                        AND A FIXED EXTERNAL LOOP GAIN  HAS BEEN  SPECIFIED
C                        OR IF THE TRANSFER FUNCTION GAIN IS FIXED AND AN EX
C                        TERNAL GAIN IS TO BE FOUND.
C         DATA CARD # 02 IN FORMAT 6F10.3.THE DESIRED VALUES FOR THE DAMPING
C                        RATIO,NATURAL FREQUENCY,REAL ROOT,ERROR   COEFFICI-
C                        ENT,SPECIFIED GAIN TRANSFER FUNCTION(IF ING IS ZERO
C                        LEAVE BLANK FIELD) AND THE SPECIFIED EXTERNAL  LOOP
C                        GAIN(IF ING IS ONE LEAVE BLANK FIELD).
C         DATA CARD # 03 IN FORMAT 2I5,THE NUMBER OF REAL ZEROS (KN) AND COM
C                        PLEX CONJUGATE ZEROS (LN) RESPECTIVELY IN THE UNCOM
C                        PENSATED TRANSFER FUNCTION.
C         DATA CARD # 04 IN FORMAT 10F8.2.THE NUMERICAL VALUE FOR THE   REAL
C                        ZEROS OF THE UNCOMPENSATED TRANSFER FUNCTION   WITH
C                        THEIR CORRESPONDING NEGATIVE SIGN IF THEY ARE  LOCA
C                        TED ON THE LEFT HALF PLANE.IF KN IN DATA CARD 03 IS
C                        EQUAL TO ZERO A BLANK CARD SHOULD BE USED.
C         DATA CARD # 05 IN FORMAT 10F8.2.THE NUMERICAL VALUE FOR THE  COMP-
```

```
C                    LEX CONJUGATE ZEROS OF THE GIVEN PLANT,G(S),ORDERED
C                    REAL AND IMAGINARY PART. EXAMPLE: (S+1-2J).(S+1+2J)
C                    WILL APPEAR AS -1.00 IN THE FIRST EIGHT COLUMNS AND
C                    2.00 SOMEWHERE BETWEEN COLUMN NINE AND SIXTEEN.  IF
C                    LN IN DATA CARD # 03 IS EQUAL TO ZERO A BLANK  CARD
C                    SHOULD BE USED.
C     DATA CARD # 06 IN FORMAT 2I5.THE NUMBER OF REAL POLES (KD) AND COM
C                    PLEX CONJUGATE POLES (LD) RESPECTIVELY IN THE UNCOM
C                    PENSATED TRANSFER FUNCTION.
C     DATA CARD # 07 SAME AS DATA CARD #04 BUT REPLACING THE WORDS ZEROS
C                    BY POLES,KN BY KD AND 03 BY 05.
C     DATA CARD # 08 SAME AS DATA CARD #05 BUT REPLACING THE WORDS ZEROS
C                    BY POLES,LM BY LD AND 03 BY 05.
C     DATA CARD # 09 IN FORMAT 10F8.3 THE COEFFICIENTS OF THE UNCOMPENSA
C                    TED NUMERATOR POLYNOMIAL ORDERED IN  DESCENDING  PO
C                    WERS OF S.(NUMBER OF COEFFICIENTS SHOULD BE M+1).
C     DATA CARD # 10 IN FORMAT 10F8.3 THE COEFFICIENTS OF THE UNCOMPENSA
C                    TED DENOMINATOR POLYNOMIAL ORDERED IN DESCENDING PO
C                    WERS OF S.(NUMBER OF COEFFICIENTS SHOULD BE N+1).
C
C
      DIMENSION ZR(15),ZCR(15),ZCI(15),PR(15),PCR(15),PCI(15),A(15),
     &Q(15),U(15),D(25),C(10),ROOTR(15),ROOTI(15),WB(15),WC(15),E(15),
     &F(15),R(20),S(20),EA(15),T(15),RO(20),EO(20)
      CALL  ERRSET (208,260,-3,1,0)
      READ(5,1)N,M,NT,INSF,ING
      READ(5,3)ZETA,OMEGA,SIGMA,ERRCO,GIN,GEX
      WRITE(6,21)
      WRITE(6,23)
      MP2=M+2
      NP1=N+1
      MP1=M+1
      IF(NT.EQ.1)GO TO 4
      READ(5,7)KN,LN
      READ(5,5)(ZR(I),I=1,KN)
      READ(5,5)(ZCR(I),ZCI(I),I=1,LN)
      READ(5,7)KD,LD
      READ(5,5)(PR(I),I=1,KD)
      READ(5,5)(PCR(I),PCI(I),I=1,LD)
      CALL COPOL(KN,ZR,LN,ZCR,ZCI,M,A)
      CALL COPOL(KD,PR,LD,PCR,PCI,N,Q)
      WRITE(6,222)ZETA
      WRITE(6,223)OMEGA
      WRITE(6,224)SIGMA
      WRITE(6,225)ERRCO
      IF(ING.EQ.0)WRITE(6,593)GEX
      IF(ING.EQ.1)WRITE(6,594)GIN
      WRITE(6,710)
      WRITE(6,8)(A(I),I=1,MP1)
      WRITE(6,720)
      WRITE(6,8)(Q(I),I=1,NP1)
      WRITE(6,226)
      WRITE(6,227)(ZR(I),I=1,KN)
      WRITE(6,228)
      WRITE(6,229)(ZCR(I),ZCI(I),I=1,LN)
      WRITE(6,230)
      WRITE(6,227)(PR(I),I=1,KD)
      WRITE(6,231)
      WRITE(6,229)(PCR(I),PCI(I),I=1,LD)
      GO TO 9
    4 READ(5,10)(A(I),I=1,MP1)
      READ(5,10)(Q(I),I=1,NP1)
      WRITE(6,710)
      WRITE(6,8)(A(I),I=1,MP1)
      WRITE(6,720)
      WRITE(6,8)(Q(I),I=1,NP1)
    9 CALL COENUM(N,M,ZETA,OMEGA,SIGMA,A,EA)
      DO 130 I=1,3
  130 D(I)=EA(I)
      DO 150 I=4,6
  150 D(I+5)=EA(I)
      DO 160 I=7,9
  160 D(I+6)=EA(I)
      CALL COEDEN(N,ZETA,OMEGA,SIGMA,Q,T)
      DO 170 I=1,3
  170 D(I+4)=T(I)
      DO 180 I=4,6
  180 C(I-3)=T(I)
  300 D(4)=-A(M+1)
      IF(Q(N+1).EQ.0.0.AND.Q(N).EQ.0.0)GO TO 18
      IF(Q(N+1).EQ.0.0)GO TO 19
      D(8)=ERRCO*Q(N+1)
      GO TO 20
   18 D(8)=ERRCO*Q(N-1)
      GO TO 20
   19 D(8)=ERRCO*Q(N)
   20 D(12)=0.0
      D(16)=0.0
      C(4)=0.0
      NA=4
      CALL SIMQ(D,C,NA,KS)
      ALFA=C(1)
      BETA=C(2)
      DELTA=C(3)
      GAMMA=C(4)
      POLE=BETA
      WRITE(6,29)
      WRITE(6,30)
      IF(INSF.EQ.1)GO TO 373
      IF(ING.EQ.1)GO TO 473
      GAINO=(GAMMA-DELTA*POLE)/GEX
      ZEROO=ALFA/(GAINO*GEX)
      TACGAO=DELTA/GAINO
      ECOO=(GAINO*ZEROO*GEX*A(M+1))/(Q(N+1)*POLE)
      WRITE(6,375)
      WRITE(6,46)GAINO,TACGAO,POLE,ZEROO
      WRITE(6,37)ECOO
      CALL COECE1(N,NO,POLE,Q,RO)
      CALL COECE2(INSF,M,MA,GAINO,TACGAO,ZEROO,POLE,GEX,A,EO)
      GO TO 374
  473 GAINO=(GAMMA-DELTA*POLE)/GIN
      ZEROO=ALFA/(GAINO*GIN)
      TACGAO=DELTA/GIN
      ECOO=(GAINO*ZEROO*GIN*A(M+1))/(Q(N+1)*POLE)
      WRITE(6,491)
      WRITE(6,475)GAINO,ZEROO,TACGAO,ZEROO
      WRITE(6,37)ECOO
      CALL COECE1(N,NO,POLE,Q,RO)
      CALL COECE2(INSF,M,MA,GIN,TACGAO,ZEROO,POLE,GAINO,A,EO)
      GO TO 374
  373 DIS=GAMMA**2-4.0*DELTA*ALFA
      IF(ING.EQ.1)GO TO 476
      IF(DIS)16,17,17
   17 GAIN1=(1.0/(2.0*GEX))*(GAMMA+SQRT(DIS))
      IF(GAIN1)13,13,14
   14 ZERO1=ALFA/(GAIN1*GEX)
      TACGA1=DELTA/GAIN1
      ECO1=(GAIN1*ZERO1*GEX*A(M+1))/(Q(N+1)*POLE)
      CALL COECE1(N,NN,POLE,Q,R)
      CALL COECE2(INSF,M,MM,GAIN1,TACGA1,ZERO1,POLE,GEX,A,S)
      WRITE(6,376)
      WRITE(6,45)
      WRITE(6,46)GAIN1,TACGA1,POLE,ZERO1
      GO TO 477
  476 IF(DIS)16,478,478
  478 GAIN1=(1.0/(2.0*GIN))*(GAMMA+SQRT(DIS))
      IF(GAIN1)13,13,479
  479 ZERO1=ALFA/(GAIN1*GIN)
      TACGA1=DELTA/GIN
      ECO1=(GAIN1*ZERO1*GIN*A(M+1))/(Q(N+1)*POLE)
      CALL COECE1(N,NN,POLE,Q,R)
      CALL COECE2(INSF,M,MM,GIN,TACGA1,ZERO1,POLE,GAIN1,A,S)
      WRITE(6,492)
      WRITE(6,45)
      WRITE(6,475)GAIN1,TACGA1,POLE,ZERO1
  477 WRITE(6,37)ECO1
C
C     COEFFICIENTS OF THE COMPENSATED CHARACTERISTIC EQUATION FOR SOL #1
C
      CALL PADD(E,NP2,R,NN,S,MM)
      WRITE(6,38)
      WRITE(6,53)(E(I),I=1,NP2)
      DO 81 J=1,NP2
   81 WB(J)=0.0
      CALL POLRT(E,WB,NP1,ROOTR,ROOTI,IER)
      WRITE(6,40)
      WRITE(6,41)
      DO 43 K=1,NP1
   43 WRITE(6,51)ROOTR(K),ROOTI(K)
      GO TO 93
   13 WRITE(6,59)
   93 IF(ING.EQ.1)GO TO 480
      GAIN2=(1.0/(2.0*GEX))*(GAMMA-SQRT(DIS))
      IF(GAIN2)48,48,15
   15 ZERO2=ALFA/(GAIN2*GEX)
      TACGA2=DELTA/GAIN2
      ECO2=(GAIN2*ZERO2*GEX*A(M+1))/(Q(N+1)*POLE)
      CALL COECE1(N,NO,POLE,Q,RO)
      CALL COECE2(INSF,M,MA,GAIN2,TACGA2,ZERO2,POLE,GEX,A,EO)
      WRITE(6,47)
      WRITE(6,46)GAIN2,TACGA2,POLE,ZERO2
      GO TO 481
  480 GAIN2=(1.0/(2.0*GIN))*(GAMMA-SQRT(DIS))
      IF(GAIN2)48,48,482
  482 ZERO2=ALFA/(GAIN2*GIN)
      TACGA2=DELTA/GIN
      ECO2=(GAIN2*ZERO2*GIN*A(M+1))/(Q(N+1)*POLE)
      CALL COECE1(N,NO,POLE,Q,RO)
      CALL COECE2(INSF,M,MA,GIN,TACGA2,ZERO2,POLE,GAIN2,A,EO)
      WRITE(6,47)
      WRITE(6,475)GAIN2,TACGA2,POLE,ZERO2
  481 WRITE(6,37)ECO2
C
C     COEFFICIENTS OF THE COMPENSATED CHARACTERISTIC EQUATION FOR SOL.#2
C
  374 CALL PADD(F,NP2,RO,NO,EO,MA)
      DO 91 J=1,NP2
   91 WC(J)=0.0
      CALL POLRT(F,WC,NP1,ROOTR,ROOTI,IER)
      WRITE(6,38)
      WRITE(6,53)(F(I),I=1,NP2)
      WRITE(6,40)
      WRITE(6,41)
      DO 61 K=1,NP1
   61 WRITE(6,51)ROOTR(K),ROOTI(K)
      WRITE(6,666)
      GO TO 777
   48 WRITE(6,49)
      GO TO 777
   16 WRITE(6,60)
    1 FORMAT(5I5)
    3 FORMAT(6F10.3)
   21 FORMAT(1H1,//,20X,'INPUT DATA')
   23 FORMAT(20X,'----------',//)
    7 FORMAT(2I5)
    5 FORMAT(10F8.2)
  222 FORMAT(//,20X,'SPECIFICATION FOR ZETA IS........................',
     &E14.5,/)
  223 FORMAT(//,20X,'SPECIFICATION FOR NATURAL FREQUENCY IS...........',
     &E14.5,/)
  224 FORMAT(//,20X,'SPECIFICATION FOR REAL ROOT IS..................',
     &E14.5,/)
  225 FORMAT(//,20X,'SPECIFICATION FOR ERROR COEFFICIENT IS...........',
     &E14.5,/)
  710 FORMAT(//,20X,'COEFFICIENTS OF NUMERATOR POLYNOMIAL IN DESCENDING
     &ORDER',/)
  720 FORMAT(//,20X,'COEFFICIENTS OF DENOMINATOR POLYNOMIAL IN DESCENDIN
     &G ORDER',/)
    8 FORMAT(20X,4E15.5)
  226 FORMAT(//,20X,'REAL ZEROS OF UNCOMPENSATED TRANSFER FUNCTION',/)
  227 FORMAT(//,20X,4E15.5,/)
  228 FORMAT(//,20X,'COMPLEX CONJUGATES ZEROS OF UNCOMPENSATED TRANSFER
     &FUNCTION',/)
  229 FORMAT(//,20X,4E15.5,/)
  230 FORMAT(//,20X,'REAL POLES OF THE UNCOMPENSATED TRANSFER FUNCTION
     &',/)
  231 FORMAT(//,20X,'COMPLEX CONJUGATES POLES OF UNCOMPENSATED TRANSFER
     &FUNCTION',/)
   10 FORMAT(10F8.3)
   29 FORMAT(1H1,//,20X,'OUTPUT DATA',/)
   30 FORMAT(20X,'----------',/)
  375 FORMAT(//,20X,'FILTER OUTSIDE INTERNAL LOOP, EXTERNAL GAIN SPECIFI
     &ED',/)
  475 FORMAT(//,20X,'EXTERNAL GAIN............K=',E15.5,//,
     &20X,'TACHOMETER GAIN..........KT=',E15.5,//,
     &20X,'COMPENSATOR POLE.........PC=',E15.5,//,
     &20X,'COMPENSATOR ZERO.........ZC=',E15.5,///)
   45 FORMAT(//,20X,'CHOICE ONE')
  376 FORMAT(//,20X,'FILTER INSIDE  INTERNAL LOOP, EXTERNAL GAIN SPECIFI
     &ED',/)
  491 FORMAT(//,20X,'FILTER OUTSIDE INTERNAL LOOP, TRANSFER GAIN SPECIFI
     &ED',/)
  492 FORMAT(//,20X,'FILTER INSIDE  INTERNAL LOOP, TRANSFER GAIN SPECIFI
     &ED',/)
   46 FORMAT(//,20X,'FORWARD GAIN...............K=',E15.5,//,
     &20X,'TACHOMETER GAIN..........KT=',E15.5,//,
     &20X,'COMPENSATOR POLE.........PC=',E15.5,//,
     &20X,'COMPENSATOR ZERO.........ZC=',E15.5,///)
   37 FORMAT(//,20X,'COMPUTED ERROR COEFFICIENT IS =',E15.5,/)
   38 FORMAT(//,20X,'COEFFICIENTS OF THE C.E. IN ASCENDING ORDER')
   53 FORMAT(//,20X,3E15.5,/)
   40 FORMAT(//,20X,'THE COMPENSATED SYSTEM ROOTS ARE',/)
   41 FORMAT(//,25X,'REAL PART',6X,'IMAGINARY',/)
   51 FORMAT(//,20X,2E15.5,/)
   59 FORMAT(//,20X,'CHOICE ONE HAS EITHER NEGATIVE OR ZERO FORWARD GAIN
     &')
   47 FORMAT(1H1,//,20X,'CHOICE TWO')
  666 FORMAT(1H1)
   49 FORMAT(//,20X,'CHOICE TWO HAS EITHER NEGATIVE OR ZERO FORWARD GAIN
     &')
   60 FORMAT(//,20X,'SYSTEM HAS COMPLEX GAIN,NO SOLUTION')
  593 FORMAT(//,20X,'SPECIFICATION FOR EXTERNAL LOOP GAIN IS...........',
     &E14.5,/)
  594 FORMAT(//,20X,'SPECIFICATION FOR TRANSFER GAIN IS................',
     &E14.5,/)
  777 STOP
      END

      SUBROUTINE COEDEN(N,Z,O,S,Q,T)
C
C     THIS SUBROUTINE COMPUTES THE VALUES OF THE DENOMINATOR POLYNOMIAL.
C
      DIMENSION T(10),U(15),Q(15)
      NP1=N+1
      U(1)=-1.
      U(2)=0.
      U(3)=1.
      DO 140  K=1,NP1
  140 U(K+3)=2.*Z*U(K+2)-U(K+1)
C
C     CASE OF DENOMINATOR POLYNOMIAL OF DEGREE TWO.
C
      T(1)=Q(N+1)*U(1)-Q(N)*U(2)*O+Q(N-1)*U(3)*O**2
      T(2)=Q(N+1)*U(2)-Q(N)*U(3)*O+Q(N-1)*U(4)*O**2
      T(3)=Q(N+1)+Q(N)*S+Q(N-1)*S**2
      T(4)=Q(N+1)*U(2)*O-Q(N)*U(3)*O**2+Q(N-1)*U(4)*O**3
      T(5)=Q(N+1)*U(3)*O-Q(N)*U(4)*O**2+Q(N-1)*U(5)*O**3
      T(6)=-Q(N+1)*S-Q(N)*S**2-Q(N-1)*S**3
      IF(N.EQ.2)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL CF DEGREE THREE
C
      T(1)=T(1)-Q(N-2)*U(4)*O**3
      T(2)=T(2)-Q(N-2)*U(5)*O**3
      T(3)=T(3)+Q(N-2)*S**3
      T(4)=T(4)-Q(N-2)*U(5)*O**4
      T(5)=T(5)-Q(N-2)*U(6)*O**4
      T(6)=T(6)-Q(N-2)*S**4
      IF(N.EQ.3)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL OF DEGREE FOUR
C
      T(1)=T(1)+Q(N-3)*U(5)*O**4
      T(2)=T(2)+Q(N-3)*U(6)*O**4
      T(3)=T(3)+Q(N-3)*S**4
      T(4)=T(4)+Q(N-3)*U(6)*O**5
      T(5)=T(5)+Q(N-3)*U(7)*O**5
      T(6)=T(6)-Q(N-3)*S**5
      IF(N.EQ.4)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL CF DEGREE FIVE
C
      T(1)=T(1)-Q(N-4)*U(6)*O**5
      T(2)=T(2)-Q(N-4)*U(7)*O**5
      T(3)=T(3)+Q(N-4)*S**5
      T(4)=T(4)-Q(N-4)*U(7)*O**6
      T(5)=T(5)-Q(N-4)*U(8)*O**6
      T(6)=T(6)-Q(N-4)*S**6
      IF(N.EQ.5)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL OF DEGREE SIX
C
      T(1)=T(1)+Q(N-5)*U(7)*O**6
      T(2)=T(2)+Q(N-5)*U(8)*O**6
      T(3)=T(3)+Q(N-5)*S**6
      T(4)=T(4)+Q(N-5)*U(8)*O**7
      T(5)=T(5)+Q(N-5)*U(9)*O**7
      T(6)=T(6)-Q(N-5)*S**7
      IF(N.EQ.6)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL OF CEGREE SEVEN
C
      T(1)=T(1)-Q(N-6)*U(8)*O**7
      T(2)=T(2)-Q(N-6)*U(9)*O**7
      T(3)=T(3)+Q(N-6)*S**7
      T(4)=T(4)-Q(N-6)*U(9)*O**8
      T(5)=T(5)-Q(N-6)*U(10)*O**8
      T(6)=T(6)-Q(N-6)*S**8
      IF(N.EQ.7)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL OF DEGREE EIGHT
C
      T(1)=T(1)+Q(N-7)*U(9)*O**8
      T(2)=T(2)+Q(N-7)*U(10)*O**8
      T(3)=T(3)+Q(N-7)*S**8
      T(4)=T(4)+Q(N-7)*U(10)*O**9
      T(5)=T(5)+Q(N-7)*U(11)*O**9
      T(6)=T(6)-Q(N-7)*S**9
      IF(N.EQ.8)RETURN
C
C     CASE OF DENOMINATOR POLYNOMIAL OF DEGREE NINE
C
      T(1)=T(1)-Q(N-8)*U(10)*O**9
      T(2)=T(2)-Q(N-8)*U(11)*O**9
      T(3)=T(3)+Q(N-8)*S**9
      T(4)=T(4)-Q(N-8)*U(11)*O**10
      T(5)=T(5)-Q(N-8)*U(12)*O**10
      T(6)=T(6)-Q(N-8)*S**10
      RETURN
      END


      SUBROUTINE COENUM(N,M,Z,O,S,A,E)
C
C     THIS SUBROUTINE COMPUTES THE VALUES OF THE NUMERATOR   POLYNOMIAL.
C
      DIMENSION E(25),U(15),A(15)
      NP1=N+1
      U(1)=-1.0
      U(2)=0.0
      U(3)=1.0
      DO 140 K=1,NP1
  140 U(K+3)=2.0*Z*U(K+2)-U(K+1)
C
C     CASE OF NUMERATOR POLYNOMIAL OF DEGREE ZERO
C
      E(1)=A(M+1)*U(1)
      E(2)=A(M+1)*U(2)
      E(3)=A(M+1)
      E(4)=A(M+1)*U(3)*O**2
      E(5)=A(M+1)*U(4)*O**2
      E(6)=A(M+1)*S**2
      E(7)=-A(M+1)*U(2)*O
      E(8)=-A(M+1)*U(3)*0
      E(9)=A(M+1)*S
      IF(M.EQ.0)RETURN
C
C     CASE OF NUMERATCR PCLYNOMIAL OF DEGREE ONE
C
      E(1)=E(1)-A(M)*U(2)*0
      E(2)=E(2)-A(M)*U(3)*0
      E(3)=E(3)+A(M)*S
      E(4)=E(4)-A(M)*U(4)*0**3
      E(5)=E(5)-A(M)*U(5)*0**3
      E(6)=E(6)+A(M)*S**3
      E(7)=E(7)+A(M)*U(3)*0**2
      E(8)=E(8)+A(M)*U(4)*0**2
      E(9)=E(9)+A(M)*S**2
      IF(M.EQ.1)RETURN
C
C     CASE OF NUMERATOR POLYNOMIAL OF DEGREE TWO
C
      E(1)=E(1)+A(M-1)*U(3)*0**2
      E(2)=E(2)+A(M-1)*U(4)*0**2
      E(3)=E(3)+A(M-1)*S**2
      E(4)=E(4)+A(M-1)*U(5)*0**4
      E(5)=E(5)+A(M-1)*U(6)*0**4
      E(6)=E(6)+A(M-1)*S**4
      E(7)=E(7)-A(M-1)*U(4)*0**3
      E(8)=E(8)-A(M-1)*U(5)*0**3
      E(9)=E(9)+A(M-1)*S**3
      IF(M.EQ.2)RETURN
C
C     CASE OF NUMERATCR PCLYNCMIAL OF DEGREE THREE
C
      E(1)=E(1)-A(M-2)*U(4)*0**3
      E(2)=E(2)-A(M-2)*U(5)*0**3
      E(3)=E(3)+A(M-2)*S**3
      E(4)=E(4)-A(M-2)*U(6)*0**5
      E(5)=E(5)-A(M-2)*U(7)*0**5
      E(6)=E(6)+A(M-2)*S**5
      E(7)=E(7)+A(M-2)*U(5)*0**4
      E(8)=E(8)+A(M-2)*U(6)*0**4
      E(9)=E(9)+A(M-2)*S**4
      IF(M.EQ.3)RETURN
C
C     CASE OF NUMERATCR PCLYNOMIAL OF DEGREE FOUR
C
      E(1)=E(1)+A(M-3)*U(5)*0**4
      E(2)=E(2)+A(M-3)*U(6)*0**4
      E(3)=E(3)+A(M-3)*S**4
      E(4)=E(4)+A(M-3)*U(7)*O**6
      E(5)=E(5)+A(M-3)*U(8)*O**6
      E(6)=E(6)+A(M-3)*S**6
      E(7)=E(7)-A(M-3)*U(6)*O**5
      E(8)=E(8)-A(M-3)*U(7)*O**5
      E(9)=E(9)+A(M-3)*S**5
      IF(M.EQ.4)RETURN
C
C     CASE OF NUMERATCR POLYNCMIAL OF DEGREE FIVE
C
      E(1)=E(1)-A(M-4)*U(6)*O**5
      E(2)=E(2)-A(M-4)*U(7)*O**5
      E(3)=E(3)+A(M-4)*S**5
      E(4)=E(4)-A(M-4)*U(8)*O**7
      E(5)=E(5)-A(M-4)*U(9)*O**7
      E(6)=E(6)+A(M-4)*S**7
      E(7)=E(7)+A(M-4)*U(7)*O**6
      E(8)=E(8)+A(M-4)*U(8)*O**6
      E(9)=E(9)+A(M-4)*S**6
      IF(M.EQ.5)RETURN
C
C     CASE OF NUMERATCR POLYNOMIAL OF DEGREE SIX
C
      E(1)=E(1)+A(M-5)*U(7)*O**6
      E(2)=E(2)+A(M-5)*U(8)*O**6
      E(3)=E(3)+A(M-5)*S**6
      E(4)=E(4)+A(M-5)*U(9)*O**8
      E(5)=E(5)+A(M-5)*U(10)*O**8
      E(6)=E(6)+A(M-5)*S**8
      E(7)=E(7)-A(M-5)*U(8)*O**7
      E(8)=E(8)-A(M-5)*U(9)*O**7
      E(9)=E(9)+A(M-5)*S**7
      IF(M.EQ.6)RETURN
C
C     CASE OF NUMERATOR POLYNOMIAL OF DEGREE SEVEN
C
      E(1)=E(1)-A(M-6)*U(8)*O**7
      E(2)=E(2)-A(M-6)*U(9)*O**7
      E(3)=E(3)+A(M-6)*S**7
      E(4)=E(4)-A(M-6)*U(10)*O**9
      E(5)=E(5)-A(M-6)*U(11)*O**9
      E(6)=E(6)+A(M-6)*S**9
      E(7)=E(7)+A(M-6)*U(9)*O**8
      E(8)=E(8)+A(M-6)*U(10)*O**8
      E(9)=E(9)+A(M-6)*S**8
      RETURN
      END

      SUBROUTINE COECE1(N,NN,P,Q,R)
```

```
C
      DIMENSION Q(25),R(25)
      NP1=N+1
      NP2=N+2
      R(1)=Q(NP1)*P
      DO 1 I=1,N
    1 R(I+1)=Q(NP1-I)*P+Q(NP2-I)
      R(NP2)=Q(1)
      NN=NP2
      RETURN
      END


      SUBROUTINE COECE2(INSF,M,MM,X,Y,Z,P,GEX,A,S)
C
      DIMENSION A(25),S(25),R(25),SS(25)
      MP1=M+1
      MP2=M+2
      MP3=M+3
      R(1)=0.0
      IF(INSF.EQ.1)GO TO 3
      R(2)=A(MP1)*X*Y*P
      GO TO 4
    3 R(2)=A(MP1)*X*Y*Z
    4 R(MP3)=A(1)*X*Y
      SS(1)=A(MP1)*X*Z*GEX
      SS(MP2)=A(1)*X*GEX
      IF(M.NE.0)GO TO 1
      CALL PADD(S,MM,R,MP3,SS,MP2)
      RETURN
    1 DO 2 I=1,M
    2 SS(I+1)=A(MP1-I)*X*Z*GEX+A(MP2-I)*X*GEX
      IF(INSF.EQ.1)GO TO 5
      DO 6 I=1,M
    6 R(I+2)=A(MP1-I)*X*Y*P+A(MP2-I)*X*Y
      GO TO 8
    5 DO 7 I=1,M
    7 R(I+2)=A(MP1-I)*X*Y*Z+A(MP2-I)*X*Y
    8 CALL PADD(S,MM,R,MP3,SS,MP2)
      RETURN
      END


      SUBROUTINE PADD(Z,IDIMZ,X,IDIMX,Y,IDIMY)
      DIMENSION Z(25),X(25),Y(25)
      NDIM=IDIMX
      IF(IDIMX-IDIMY)10,20,20
   10 NDIM=IDIMY
```

```
20 IF(NDIM)90,90,30
30 DO 80 I=1,NDIM
   IF( I-IDIMX)40,40,60
40 IF( I-IDIMY)50,50,70
50 Z( I )=X( I )+Y( I )
   GO TO 80
60 Z( I )=Y( I )
   GO TO 80
70 Z( I )=X( I )
80 CONTINUE
90 IDIMZ=NDIM
   RETURN
   END


   SUBROUTINE SIMQ(A,B,N,KS)
   DIMENSION A(1),B(1)
   TOL=0.0
   KS=0
   JJ=-N
   DO 65 J=1,N
   JY=J+1
   JJ=JJ+N+1
   BIGA=0
   IT=JJ-J
   DO 30 I=J,N
   IJ=IT+I
   IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
20 BIGA=A(IJ)
   IMAX=I
30 CONTINUE
   IF(ABS(BIGA)-TOL)35,35,40
35 KS=1
   RETURN
40 I1=J+N*( J-2)
   IT=IMAX-J
   DO 50 K=J,N
   I1=I1+N
   I2=I1+IT
   SAVE=A( I1)
   A( I1)=A( I2)
   A( I2)=SAVE
50 A( I1)=A( I1)/BIGA
   SAVE=B( IMAX)
   B( IMAX)=B( J)
   B( J)=SAVE/BIGA
   IF( J-N) 55,70,55
55 IQS=N*( J-1)
```

```
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END

      SUBROUTINE PCLMLT (NA,NB,NC,A,B,C)
C
      DIMENSION A(1),B(1),C(1)
      NC=NA+NB
      NAP2=NA+2
      NBP2=NB+2
      NCP1=NC+1
      DO 6 I=NAP2,NCP1
    6 A(I)=0.0
      DO 7 I=NBP2,NCP1
    7 B(I)=0.0
    8 C(1)=A(1)*B(1)
      IF(NC.LT.1)RETURN
    9 C(2)=A(1)*B(2)+A(2)*B(1)
      IF(NC.LT.2)RETURN
      DO 12 I=3,NCP1
   10 C(I)=A(1)*B(I)
      DO 12 J=2,I
   11 K=I-J+1
   12 C(I)=C(I)+A(J)*B(K)
      RETURN
      END

      SUBROUTINE CCPOL (K,R,L,P,Q,N,A)
C
```

```
C
      DIMENSION R(1),P(1),Q(1),A(1),B(50),C(50)
      A(1)=1.0
      B(1)=1.0
   10 IF(K.EQ.0)GO TO 18
   11 A(2)=-R(1)
   12 IF(K.EQ.1)GO TO 18
      N=1
   13 DO 17 J=2,K
   14 B(2)=-R(J)
      CALL POLMLT(N,1,NC,A,B,C)
   15 N=N+1
      NP1=N+1
      DO 16 I=2,NP1
   16 A(I)=C(I)
   17 CONTINUE
   18 IF(L.EQ.0)RETURN
      N=K
      DO 23 J=1,L
   19 B(2)=-2.0*P(J)
      B(3)=P(J)*P(J)+Q(J)*Q(J)
   20 CALL POLMLT(N,2,NC,A,B,C)
   21 N=N+2
      NP1=N+1
      DO 22 I=2,NP1
   22 A(I)=C(I)
   23 CONTINUE
      RETURN
      END
      ************FIVE SPECIFICATIONS DESIGN PROGRAM(FISDEP)************
C
C
C     THIS PROGRAM USES ALGEBRAIC METHODS TO DESIGN A CONTROL SYSTEM WHI
C     CH MUST SATISFY THREE,FOUR OR FIVE SPECIFICATIONS. IT IS   ASSUMED
C     THAT THE DIFFERENTIAL EQUATIONS OF THE UNCOMPENSATED SYSTEM    ARE
C     KNOWN AND CAN BE EXPRESSED IN TERMS OF A TRANSFER FUNCTION. THE DE
C     SIGN PROCEDURE IS BASED ON THE USE OF THE CHARACTERISTIC  EQUATION
C     FORMED BY THE UNCOMPENSATED SYSTEM AND A SET OF COMPENSATOR   DEVI
C     CES. THE SPECIFICATIONS THAT MAY BE USED ARE:DAMPING RATIO,NATURAL
C     FREQUENCY,CONSTANT ERROR COEFFICIENT,BANDWIDTH AND    SENSITIVITY.
C     THEY MAY BE USED ALL AT THE SAME TIME(USING COMBINATION OF  SINGLE
C     SECTION OF FILTER IN CASCADE,AND ACCELERATION AND TACHOMETER  FEED
C     BACK AS COMPENSATOR DEVICES) OR IN THE FOLLOWING ALTERNATIVES:
C     1. A COMBINATION OF DAMPING RATIO,NATURAL FREQUENCY AND  SENSITIVI
C        TY(USING TACHOMETER AND ACCELERATION FEEDBACK AS  COMPENSATORS)
C     2. A COMBINATION OF DAMPING RATIO, NATURAL FREQUENCY,  SENSITIVITY
C        AND BANDWIDTH(USING SINGLE SECTION OF FILTER IN CASCADE AND  TA
CC       CHOMETER OR ACCELERATION FEEDBACK AS COMPENSATORS)
```

```
C      3. A COMBINATION OF DAMPING RATIO,NATURAL FREQUENCY,BANDWIDTH  AND
C         ERROR COEFFICIENT(SAME COMPENSATORS AS ALTERNATIVE 2)
C      SINCE THE SOLUTION IS BASED ON THE CHARACTERISTIC EQUATION OF  THE
C      COMPENSATED SYSTEM THERE IS NO  RESTRICTION FOR CHOOSING A PARTICU
C      LAR COMBINATION OF COMPENSATOR DEVICES PROVIDED THAT THE TOTAL NUM
C      BER OF UNKNOWN PARAMETERS OF THE CHARACTERISTIC EQUATION    EQUALS
C      THE NUMBER OF SPECIFICATIONS(IN THIS PROGRAM THE SYSTEM FORWARD GA
C      IN WILL BE CONSIDERED AN UNKNOWN PARAMETER)
C      THE INPUT TO THE PROGRAM IS THE FOLLOWING:
C      A. A SUBROUTINE(CALLED CALCE1)WITH THE COEFFICIENTS OF THE  CHARAC
C         TERISTIC EQUATION AS A FUNCTION OF THE UNKNOWN PARAMETERS. THIS
C         SUBROUTINE MUST REPLACE THE EXAMPLE GIVEN HERE.
C      B. DATA CARD #01 WRITTEN IN FORMAT 2F10.2 WITH THE  SPECIFICATIONS
C         FOR DAMPING RATIO AND NATURAL FREQUENCY.
C      C. DATA CARD #02 WRITTEN IN FORMAT 2I5 WITH THE ORDER OF THE   CHA
C         RACTERISTIC EQUATION AND THE INTEGER:
C         1 IF ALTERNATIVE ONE IS USED.
C         2 IF ALTERNATIVE TWO IS USED.
C         3 IF ALTERNATIVE THREE IS USED.
C         4 IF ALTERNATIVE FOUR(ALL FIVE SPECIFICATIONS) IS USED.
C      D. DATA CARD #03 WRITTEN IN FORMAT 9F8.2 WITH A SET OF GUESSED  VA
C         LUES FOR THE UNKNOWN PARAMETERS,A TERMINATION CRITERION AND  RE
C         MAINING SPECIFICATIONS,THAT IS,IF FORWARD GAIN,TACHOMETER GAIN,
C         ACCELERATION GAIN,COMPENSATOR ZERO,COMPENSATOR POLE,TERMINATION
C         CRITERION AND SENSITIVITY,BANDWIDTH AND ERROR COEFFICIENT SPECI
C         FICATION ARE CALLED:X,Y,Z,P,R,EPS,SE,B AND EC RESPECTIVELY THEN
C         THE THIRD DATA CARD SHOULD BE:
C         IN FORMAT 9F8.2 VALUES FOR X,Y,Z,EPS AND SE: OR X,Y,P,R, EPS, B
C         AND SE: OR X,Y,P,R,EPS,B AND EC: OR X,Y,Z,P,R,EPS,SE,B  AND  EC
C         IF ALTERNATIVE ONE,TWO,THREE OR FOUR RESPECTIVELY IS USED.
C      E. A DATA STATEMENT WITH THE ORDER OF THE CHARACTERISTIC  EQUATION
C         AND THE SPECIFICATIONS SHOULD REPLACE THE PRESENT ONE IN    THE
C         FUNCTION SUB-PROGRAM H,G,Q OR F IF ALTERNATIVE ONE,TWO,THREE OR
C         FOUR RESPECTIVELY IS USED.
C      F. NEW STATEMENTS FOR BCN AND BCD SHOULD BE DERIVED FROM THE  ABSO
C         LUTE VALUE OF THE CLOSED LOOP TRANSFER FUNCTION,IN TERMS OF THE
C         UNKNOWN COEFFICIENTS,AND REPLACE THE PRESENT ONES IN THE   MAIN
C         PROGRAM.LIKEWISE IF ALTERNATIVES 1,3 OR 4 ARE BEING USED   THEN
C         STATEMENT # 22 OF FUNCTIONS G,Q OR F RESPECTIVELY SHOULD BE  MO
C         DIFIED ACCORDINGLY.
C      G. NEW STATEMENT FOR ECO SHOULD BE DERIVED FROM THE ERROR COEFFICI
C         ENT EQUATION,IN TERMS OF THE UNKNOWNS,AND REPLACE THE PRESENT O
C         NE IN THE MAIN PROGRAM.LIKEWISE IF ALTERNATIVE TWO OR FOUR  ARE
C         USED STATEMENT # 23 IN FUNCTION Q OR # 24 IN FUNCTION F  RESPEC
C         TIVELY,SHOULD BE MODIFIED.
C      AS AN ALTERNATIVE THE PROGRAM MAY BE USED ALTOGETHER CHANGING DATA
C      CARD # 02 TO READ ONLY THE ORDER OF THE CHARACTERISTIC    EQUATION
C      AND MAKING A "DO LOOP"(WITH NCD=1,4) FROM JUST AFTER THE STATEMENT
```

```
C     READ(5,940) TO JUST BEFORE THE STOP STATEMENT.IN THIS CASE IT  SHO
C     ULD BE NOTED THAT FOUR SETS OF DATA CARDS 2 AND 3 SHOULD BE  FURNI
C     SHED.THE PROGRAM AS PRESENTED HERE USES THIS LAST ALTERNATIVE  FOR
C     CESIGNING COMPENSATORS FOR A THIRD ORDER SYSTEM.
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL Q,F,G,H
      DIMENSION W(15),A(15),ROOTR(15),ROOTI(15),U(15),UD(15),AD(15)
      READ(5,940)ZE,O
      DO 222 NCD=1,4
      READ(5,444)N
      WRITE(6,10)
      WRITE(6,111)
      GO TO (445,446,447,448),NCD
  445 READ(5,333)X,Y,Z,EPS,SE
      WRITE(6,555)X,Y,Z,EPS
      WRITE(6,450)
      WRITE(6,451)
      WRITE(6,15)SE
      GO TO 449
  446 READ(5,333)X,Y,Z,P,EPS,SE,B
      WRITE(6,555)X,Y,Z,P,EPS
      WRITE(6,451)
      WRITE(6,16)B
      WRITE(6,15)SE
      GO TO 449
  447 READ(5,333)X,Y,Z,P,EPS,B,EC
      WRITE(6,555)X,Y,Z,P,EPS
      WRITE(6,452)
      WRITE(6,14)EC
      WRITE(6,16)B
      GO TO 449
  448 READ(5,333)X,Y,Z,P,R,EPS,SE,B,EC
      WRITE(6,555)X,Y,Z,P,R,EPS
      WRITE(6,15)SE
      WRITE(6,16)B
      WRITE(6,14)EC
  449 WRITE(6,12)ZE
      WRITE(6,13)O
      CALL CHEVY(ZE,N,U,UD)
      WRITE(6,17)
      WRITE(6,18)
      WRITE(6,720)
      WRITE(6,721)
      GO TO (500,501,502,503),NCD
  500 CALL SOLON(H,X,Y,Z,EPS,1,LE)
      CALL CALCE1(X,Y,0.0,0.0,Z,A,AD)
      WRITE(6,2)X
      WRITE(6,3)Y
      WRITE(6,4)Z
      WRITE(6,9)LE
      GO TO 504
  501 CALL SOLNS(G,X,Y,Z,P,EPS,1,LO)
      CALL CALCE1(X,Y,Z,P,0.0,A,AD)
      WRITE(6,2)X
      WRITE(6,3)Y
      WRITE(6,5)P
      WRITE(6,6)Z
      WRITE(6,9)LO
      GO TO 504
  502 CALL SOLNS(Q,X,Y,Z,P,EPS,1,LU)
      CALL CALCE1(X,0.0,Z,P,Y,A,AD)
      WRITE(6,2)X
      WRITE(6,4)Y
      WRITE(6,5)P
      WRITE(6,6)Z
      WRITE(6,9)LU
      GO TO 504
  503 CALL SOLNLS(F,X,Y,Z,P,R,EPS,1,LP)
      CALL CALCE1(X,Y,Z,P,R,A,AD)
      WRITE(6,2)X
      WRITE(6,3)Y
      WRITE(6,4)R
      WRITE(6,5)P
      WRITE(6,6)Z
      WRITE(6,9)LP
  504 NP1=N+1
      CALL SENSIT(O,N,A,AD,U,UD,DET,DETKZ)
      SE=-(X*DETKZ)/(ZE*DET)
      IF(NCD.EQ.1)GO TO 941
      ECO=(X*Z)/(A(2)-X)
      BV=DSQRT((X*Z)**2+(X*B)**2)
      BR=DSQRT((A(5)*B**4-A(3)*B**2+A(1))**2+(A(2)*B-A(4)*B**3)**2)
      BC=BV/BR
      GO TO 942
  941 ECO=X/A(2)
  942 DO 1 I=1,NP1
    1 W(I)=0.
      CALL DPOLRT(A,W,N,ROOTR,ROOTI,IER)
      WRITE(6,722)
      WRITE(6,723)
      IF(NCD.NE.1)WRITE(6,7)BC
      WRITE(6,8)SE
      WRITE(6,42)ECO
      WRITE(6,724)
      WRITE(6,725)
      WRITE(6,38)
      WRITE(6,53)(A(I),I=1,NP1)
      WRITE(6,40)
      WRITE(6,41)
      DO 43 K=1,N
   43 WRITE(6,51)ROOTR(K),ROOTI(K)
    2 FORMAT(//,20X,'FORWARD GAIN IS',10X,E15.5,/)
    3 FORMAT(//,20X,'TACHOMETER GAIN IS',7X,E15.5,/)
    4 FORMAT(//,20X,'ACCELERATION GAIN IS',5X,E15.5,/)
    5 FORMAT(//,20X,'COMPENSATOR POLE IS',6X,E15.5,/)
    6 FORMAT(//,20X,'COMPENSATOR ZERO IS',6X,E15.5,/)
    7 FORMAT(//,20X,'ABS.VALUE OF CLTF AT B IS',E15.5,/)
    8 FORMAT(//,20X,'SENSITIVITY IS',11X,E15.5,/)
    9 FORMAT(//,20X,'NUMBER OF ITERATIONS IS',12X,I5,/)
   10 FORMAT(1H1,//,20X,'INPUT DATA')
   12 FORMAT(//,20X,'SPECIFICATION FOR DAMPING RATIO IS',4X,E15.5,/)
   13 FORMAT(//,20X,'SPECIFICATION FOR NATURAL FREQUENCY IS',E15.5,/)
   14 FORMAT(//,20X,'SPECIFICATION FOR ERROR COEFFICIENT IS',E15.5,/)
   15 FORMAT(//,20X,'SPECIFICATION FOR ROOT SENSITIVITY IS',1X,E15.5,/)
   16 FORMAT(//,20X,'SPECIFICATION FOR BANDWIDTH IS',8X,E15.5,/)
   17 FORMAT(//,20X,'OUTPUT DATA')
   18 FORMAT(20X,'****** ****',//)
   38 FORMAT(//,20X,'COEFFICIENTS OF THE C.E. IN ASCENDING ORDER')
   40 FORMAT(//,20X,'THE COMPENSATED SYSTEM ROOTS ARE',/)
   41 FORMAT(//,25X,'REAL PART',6X,'IMAGINARY',/)
   42 FORMAT(//,20X,'ERROR COEFFICIENT IS',5X,E15.5,/)
   51 FORMAT(//,20X,2E15.5,/)
   53 FORMAT(//,20X,3E15.5,/)
  111 FORMAT(20X,'***** ****')
  333 FORMAT(9F8.2)
  444 FORMAT(I5)
  450 FORMAT(//,20X,'SPECIFICATION FOR BANDWIDTH IS',12X,'NONE',/)
  451 FORMAT(//,20X,'SPECIFICATION FOR ERROR COEFFICIENT IS',4X,'NONE',/
     &)
  452 FORMAT(//,20X,'SPECIFICATION FOR SENSITIVITY IS',10X,'NONE',/)
  555 FORMAT(//,20X,'INITIAL VALUES FOR UNKNOWN PARAMETERS ARE',//,20X,
     &3E15.5,//,20X,3E15.5,//)
  720 FORMAT(//,20X,'I.UNKNOWN PARAMETERS')
  721 FORMAT(20X,'--------- ----------',//)
  722 FORMAT(1H1,//,20X,'II.CALCULATED SPECIFICATIONS')
  723 FORMAT(20X,'------------- --------------',//)
  724 FORMAT(//,20X,'III.STABILITY INFORMATION')
  725 FORMAT(20X,'------------- -----------',//)
  940 FORMAT(2F10.2)
  222 CONTINUE
      STOP
      END
      SUBROUTINE DPOLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
      IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
      IFIT=0
      N=M
      IER=0
      IF(XCOF(N+1))10,25,10
   10 IF(N) 15,15,32
   15 IER=1
   20 RETURN
   25 IER=4
      GO TO 20
   30 IER=2
      GO TO 20
   32 IF(N-36) 35,35,30
   35 NX=N
      NXX=N+1
      N2=1
      KJ1 = N+1
      DO 40 L=1,KJ1
      MT=KJ1-L+1
   40 COF(MT)=XCOF(L)
   45 XO=.00500101
      YO=0.01000101
      IN=0
   50 X=XO
      XO=-10.0*YO
      YO=-10.0*X
      X=XO
      Y=YO
      IN=IN+1
      GO TO 59
   55 IFIT=1
      XPR=X
      YPR=Y
   59 ICT=0
   60 UX=0.0
      UY=0.0
      V =0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U) 65,130,65
   65 DO 70 I=1,N
      L =N-I+1
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+COF(L  )*XT2
      V=V+COF(L  )*YT2
      FI=I
      UX=UX+FI*XT*COF(L )
      UY=UY-FI*YT*COF(L )
      XT=XT2
   70 YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ) 75,110,75
   75 DX=( V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+ V*UX)/SUMSQ
      Y=Y+DY
   78 IF(DABS(DY)+DABS(DX)-1.0E-05) 100,80,80
   80 ICT=ICT+1
      IF(ICT-500) 60,85,85
   85 IF(IFIT)100,90,100
   90 IF(IN-5) 50,95,95
   95 IER=3
      GO TO 20
  100 DO 105 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
  105 COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF(IFIT) 120,55,120
  110 IF(IFIT) 115,50,115
  115 X=XPR
      Y=YPR
  120 IFIT=0
  122 IF(DABS(Y/X)-1.0E-04) 135,125,125
  125 ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GO TO 140
  130 X=0.0
      NX=NX-1
      NXX=NXX-1
  135 Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
  140 COF(2)=COF(2)+ALPHA*COF(1)
  145 DO 150 L=2,N
```

```
150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
155 ROOTI(N2)=Y
    ROOTR(N2)=X
    N2=N2+1
    IF(SUMSQ) 160,165,160
160 Y=-Y
    SUMSQ=0.0
    GO TO 155
165 IF(N) 20,20,45
    END


    SUBROUTINE SIMQ(A,B,N,KS)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION A(1),B(1)
    TOL=0.0
    KS=0
    JJ=-N
    DO 65 J=1,N
    JY=J+1
    JJ=JJ+N+1
    BIGA=0
    IT=JJ-J
    DO 30 I=J,N
    IJ=IT+I
    IF(DABS(BIGA)-DABS(A(IJ)))20,30,30
 20 BIGA=A(IJ)
    IMAX=I
 30 CONTINUE
    IF(DABS(BIGA)-TOL)35,35,40
 35 KS=1
    RETURN
 40 I1=J+N*(J-2)
    IT=IMAX-J
    DO 50 K=J,N
    I1=I1+N
    I2=I1+IT
    SAVE=A(I1)
    A(I1)=A(I2)
    A(I2)=SAVE
 50 A(I1)=A(I1)/BIGA
    SAVE=B(IMAX)
    B(IMAX)=B(J)
    B(J)=SAVE/BIGA
    IF(J-N) 55,70,55
 55 IQS=N*(J-1)
    DO 65 IX=JY,N
    IXJ=IQS+IX
```

```
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END


      SUBROUTINE SOLNLS(F,X,Y,Z,P,R,EPS,MZ,L)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     THIS SUBROUTINE USES ROBINSON METHOD TO SOLVE A SYSTEM OF FIVE
C     EQUATIONS WITH FIVE UNKNOWNS.
C     IF MZ IS SET EQUAL TO ZERO PRINT OUT EVERY FIVE ITERATIONS WILL
C     BE OBTAINED
C
      DIMENSION A(5,5),D(5,5),B(5),E(5),AA(25),KK(1000)
      DO 2000 I=1,1000
 2000 KK(I)=0
      KK(1)=1
      KA=1
      DO 1000 I=1,1000,5
      KK(I)=5*KA
      KA=KA+1
 1000 CONTINUE
      DO 20 K=1,1000
      IF(K.EQ.KK(K).AND.MZ.EQ.0)WRITE(6,39)K,X,Y,Z,P,R
   39 FORMAT(//,20X,'ON IERATION #',I4,'THE UNKNOWN VALUES ARE',5E15.5)
      L=K
      IF(K.GT.1)GO TO 9
C
C     INITIAL VALUES FOR THE DIFFERENCES X(K)-X(K-1).
C
      DX=0.1
      DY=0.1
      DZ=0.1
      DP=0.1
      DR=0.1
    9 U=DSQRT(DX*DX+DY*DY)
      V=DSQRT(DX*DX+DY*DY+DZ*DZ)
      W=DSQRT(DX*DX+DY*DY+DZ*DZ+DP*DP)
      S=DSQRT(DX*DX+DY*DY+DZ*DZ+DP*DP+DR*DR)
      D(1,1)=S*DY/U
      D(1,2)=S*DX*DZ/(V*U)
      D(1,3)=S*DX*DP/(W*V)
      D(1,4)=DX*DR/W
      D(1,5)=-DX
      D(2,1)=-S*DX/U
      D(2,2)=S*DY*DZ/(V*U)
      D(2,3)=S*DY*DP/(W*V)
      D(2,4)=DY*DR/W
      D(2,5)=-DY
      D(3,1)=0.0
      D(3,2)=-S*U/V
      D(3,3)=S*DZ*DP/(W*V)
      D(3,4)=DZ*DR/W
      D(3,5)=-DZ
      D(4,1)=0.0
      D(4,2)=0.0
      D(4,3)=-V*S/W
      D(4,4)=DP*DR/W
      D(4,5)=-DP
      D(5,1)=0.0
      D(5,2)=0.0
      D(5,3)=0.0
      D(5,4)=-W
      D(5,5)=-DR
   11 DO 12 I=1,5
   12 B(I)=-F(I,X,Y,Z,P,R)
      IF(DABS(B(1)).LT.EPS.AND.DABS(B(2)).LT.EPS.AND.DABS(B(3)).LT.EPS.
     &AND.DABS(B(4)).LT.EPS.AND.DABS(B(5)).LT.EPS)RETURN
      DO 14 I=1,5
      DO 14 J=1,5
   14 A(I,J)=F(I,X+D(1,J),Y+D(2,J),Z+D(3,J),P+D(4,J),R+D(5,J))+B(I)
      DO 15 M=1,5
   15 AA(M)=A(M,1)
      IA=1
      DO 16 M=6,10
      AA(M)=A(IA,2)
      IA=IA+1
   16 CONTINUE
      IB=1
      DO 17 M=11,15
      AA(M)=A(IB,3)
      IB=IB+1
   17 CONTINUE
      IC=1
      DO 32 M=16,20
      AA(M)=A(IC,4)
      IC=IC+1
   32 CONTINUE
      ID=1
      DO 33 M=21,25
      AA(M)=A(ID,5)
      ID=ID+1
   33 CONTINUE
      CALL SIMQ(AA,B,5,KS)
      DO 34 I=1,5
   34 E(I)=B(I)
      DX=0.0
      DY=0.0
      DZ=0.0
      DP=0.0
      DR=0.0
      DO 18 J=1,5
      DX=DX+D(1,J)*E(J)
      DY=DY+D(2,J)*E(J)
      DZ=DZ+D(3,J)*E(J)
      DP=DP+D(4,J)*E(J)
   18 DR=DR+D(5,J)*E(J)
      X=X+DX
      Y=Y+DY
      Z=Z+DZ
      P=P+DP
      R=R+DR
   20 CONTINUE
   21 L=0
      RETURN
      END
```

```
      SUBROUTINE SOLNS(G,X,Y,Z,P,EPS,MZ,L)
      IMPLICIT REAL*8(A-H,O-Z)
C
C
C     THIS SUBROUTINE USES ROBINSON'S METHOD TO SOLVE A SYSTEM OF FOUR
C     NON-LINEAR EQUATIONS WITH FOUR UNKNOWNS.
C     IF MZ IS SET EQUAL TO ZERO PRINT OUT EVERY FIVE ITERATIONS WILL
C     BE OBTAINED
C
      DIMENSION A(4,4),D(4,4),B(4),E(4),AA(16),KK(1000)
      DO 2000 I=1,1000
```

```
 2000 KK(I)=0
      KK(1)=1
      KA=1
      DO 1000 I=5,1000,5
      KK(I)=5*KA
      KA=KA+1
 1000 CONTINUE
      DO 20 K=1,1000
      IF(K.EQ.KK(K).AND.MZ.EQ.0)WRITE(6,39)K,X,Y,Z,P
   39 FORMAT(//,20X,'ON IERATION #',I4,'THE UNKNOWN VALUES ARE',4E15.5)
      L=K
      IF(K.GT.1)GO TO 9
      DO 7 I=1,4
      DO 7 J=1,4
    7 D(I,J)=0.0
      DO 8 I=1,4
    8 D(I,I)=0.1
      GO TO 11
    9 U=DSQRT(DX*DX+DY*DY)
      V=DSQRT(DX*DX+DY*DY+DZ*DZ)
      T=DSQRT(DX*DX+DY*DY+DZ*DZ+DP*DP)
      D(1,1)=DY*T/U
      D(1,2)=(DX*DZ*T)/(V*U)
      D(1,3)=DX*DP/V
      D(1,4)=-DX
      D(2,1)=-DX*T/U
      D(2,2)=(DY*DP*T)/(V*U)
      D(2,3)=DY*DP/V
      D(2,4)=-DY
      D(3,1)=0.0
      D(3,2)=-U*T/V
      D(3,3)=DZ*DP/V
      D(3,4)=-DZ
      D(4,1)=0.0
      D(4,2)=0.0
      D(4,3)=-V
      D(4,4)=-DP
   11 DO 12 I=1,4
   12 B(I)=G(I,X,Y,Z,P)
      IF(DABS(B(1)).LT.EPS.AND.DABS(B(2)).LT.EPS.AND.DABS(B(3)).LT.EPS.
     &AND.DABS(B(4)).LT.EPS)RETURN
      DO 14 I=1,4
      DO 14 J=1,4
   14 A(I,J)=G(I,X+D(1,J),Y+D(2,J),Z+D(3,J),P+D(4,J))-B(I)
      DO 15 M=1,4
   15 AA(M)=A(M,1)
      IA=1
      DO 16 M=5,8
      AA(M)=A(IA,2)
      IA=IA+1
   16 CONTINUE
      IB=1
      DO 17 M=9,12
      AA(M)=A(IB,3)
      IB=IB+1
   17 CONTINUE
      IC=1
      DO 32 M=13,16
      AA(M)=A(IC,4)
      IC=IC+1
   32 CONTINUE
      CALL SIMQ(AA,B,4,KS)
      DO 34 I=1,4
   34 E(I)=B(I)
      DX=0.0
      DY=0.0
      DZ=0.0
      DP=0.0
      DO 18 J=1,4
      DX=DX-D(1,J)*E(J)
      DY=DY-D(2,J)*E(J)
      DZ=DZ-D(3,J)*E(J)
   18 DP=DP-D(4,J)*E(J)
      X=X+DX
      Y=Y+DY
      Z=Z+DZ
      P=P+DP
   20 CONTINUE
   21 L=0
      RETURN
      END


      SUBROUTINE SOLCN(H,X,Y,Z,EPS,MZ,L)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(3,3),D(3,3),B(3),E(3),AA(9),KK(1000)
C
C     IF MZ IS SET EQUAL TO ZERO PRINT OUT EVERY FIVE ITERATIONS WILL
C     BE OBTAINED
C
      DO 2000 I=1,1000
 2000 KK(I)=0
      KK(1)=1
      KA=1
      DO 1000 I=5,1000,5
      KK(I)=5*KA
      KA=KA+1
 1000 CONTINUE
      DO 20 K=1,1000
      IF(K.EQ.KK(K).AND.MZ.EQ.0)WRITE(6,39)K,X,Y,Z
   39 FORMAT(//,20X,'ON IERATION #',I4,'THE UNKNOWN VALUES ARE',3E15.5)
      L=K
      IF(K.GT.1)GO TO 9
      DO 7 I=1,3
      DO 7 J=1,3
    7 D(I,J)=0.0
      DO 8 I=1,3
    8 D(I,I)=0.1
      GO TO 11
    9 R=DSQRT(DX*DX+DY*DY+DZ*DZ)
      S=DSQRT(DX*DX+DY*DY)
   10 D(1,1)=R*DY/S
      D(1,2)=DX*DZ/S
      D(1,3)=-DX
      D(2,1)=-R*DX/S
      D(2,2)=DY*DZ/S
      D(2,3)=-DY
      D(3,1)=0.0
      D(3,2)=-S
      D(3,3)=-DZ
   11 DO 12 I=1,3
   12 B(I)=H(I,X,Y,Z)
      IF(DABS(B(1)).LT.EPS.AND.DABS(B(2)).LT.EPS.AND.DABS(B(3)).LT.EPS)
     &RETURN
      DO 13 I=1,3
      DO 13 J=1,3
   13 A(I,J)=H(I,X+D(1,J),Y+D(2,J),Z+D(3,J))-B(I)
      DO 35 M=1,3
   35 AA(M)=A(M,1)
      IA=1
      DO 36 M=4,6
      AA(M)=A(IA,2)
      IA=IA+1
   36 CONTINUE
      IB=1
      DO 37 M=7,9
      AA(M)=A(IB,3)
      IB=IB+1
   37 CONTINUE
      CALL SIMQ(AA,B,3,KS)
      DO 34 I=1,3
   34 E(I)=B(I)
   16 DX=0.0
      DY=0.0
      DZ=0.0
      DO 17 J=1,3
      DX=DX-D(1,J)*E(J)
      DY=DY-D(2,J)*E(J)
   17 DZ=DZ-D(3,J)*E(J)
   18 X=X+DX
      Y=Y+DY
      Z=Z+DZ
   20 CONTINUE
   21 L=0
      RETURN
      END


      SUBROUTINE CHEVY(ZE,N,U,UD)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     THIS SUBROUTINE USES A RECURRENCE RELATION TO CALCULATE CHEBYCHEV'S
C     FUNCTIONS  AND ALSO CALCULATES THE FIRST DERIVATIVE OF THE FIRST 10
C     FUNCTIONS.
C
      DIMENSION U(15),UD(15)
      NP1=N+1
      U(1)=-1.0
      U(2)=0.0
      U(3)=1.0
      DO 140 K=1,NP1
  140 U(K+3)=2.0*ZE*U(K+2)-U(K+1)
      DO 141 K=1,3
  141 UD(K)=0.0
      UD(4)=2.0
      UD(5)=8.0*ZE
      UD(6)=24.0*ZE**2-4.0
      UD(7)=64.0*ZE**3-24.0*ZE
      UD(8)=160.0*ZE**4-96.0*ZE**2+6.0
      UD(9)=384.0*ZE**5-320.0*ZE**3+48.0*ZE
      UD(10)=896.0*ZE**6-960.0*ZE**4+240.0*ZE**2-8.0
      RETURN
      END


      SUBROUTINE SENSIT(C,N,A,AD,U,UD,DET,DETKZ)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(15),AD(15),U(15),UD(15)
C
C     THIS SUBROUTINE USES KOKOTOVIC AND SILJAK EQUATIONS TO   CALCULATE
C     THE DETERMINANTS REQUIRED TO FIND THE ROOT SENSITIVITY OF AN N OR-
C     DER CHARACTERISTIC EQUATION. THE PARAMETERS DET AND DETKZ ARE  THE
```

```
C       OUTPUT OF THE SUBROUTINE AND THE CORRESPONDING INPUT ARE:
C       1. THE UNDAMPED NATURAL FREQUENCY (O).
C       2. THE ORDER OF THE CHARACTERISTIC EQUATION (N).
C       3. THE N+2 ARRAYS U AND UD CONTAINING THE CHEBYCHEV'S FUNCTIONS OF
C          FIRST KIND AND ITS FIRST DERIVATIVES RESPECTIVELY.
C       4. THE N+1 ARRAYS CCNTAINING THE COEFFICIENTS OF THE  CHARACTERIS-
C          TIC EQUATION AND ITS PARTIAL DERIVATIVES WITH RESPEC TC THE SYS
C          TEM GAIN RESPECTIVELY.
C
   23 A1=0.
      DO11 K=1,N
   11 A1=A1+(-1.0)**K*K*O**K*U(K+1)*A(K+1)
      A2=0.
      DO22 K=1,N
   22 A2=A2+(-1.)**K*K*O**K*U(K+2)*A(K+1)
      B1=A(1)*UD(1)
      DO33 K=1,N
   33 B1=B1+(-1.)**K*O**K*UD(K+1)*A(K+1)
      B2=A(1)*UD(2)
      DO 44 K=1,N
   44 B2=B2+(-1.)**K*O**K*UD(K+2)*A(K+1)
      C1=AD(1)*U(1)
      DO55 K=1,N
   55 C1=C1+(-1.)**K*C**K*U(K+1)*AD(K+1)
      C2=AD(1)*U(2)
      DO66 K=1,N
   66 C2=C2+(-1.)**K*C**K*U(K+2)*AD(K+1)
      DET=A1*B2-A2*B1
      DETKZ=A1*C2-A2*C1
      RETURN
      END


      SUBROUTINE CALCE1(X,Y,Z,P,R,A,AD)
C
C       THIS SUBROUTINE IS SUPPLIED BY THE DESIGNER AND CONTAINS THE COEF-
C       FICIENTS OF THE CLCSED LOOP TRANSFER FUNCTION IN TERMS OF THE UNK-
C       NCWN PARAMETERS X,Y,Z,P AND R. THE EXAMPLE GIVEN HERE  CORRESPONDS
C       TC A THIRD ORDER SYSTEM WHOSE TRANSFER FUNCTION IS K/S3+7.5S2+3.5S
C       AND WHICH IS DESIRED TO COMPENSATE WITH A CACADED SINGLE   SECTION
C       OF FILTER AND TACHOMETER AND ACCELERATION FEECBACK COMPENSATORS IN
C       SIDE A NEGATIVE UNITY FEEDBACK LOOP.
C       THE PARAMETERS X,Y,R,P AND Z CORRESPOND TO FORWARD GAIN,TACHOMETER
C       GAIN,ACCELERATION GAIN,COMPENSATOR POLE  AND COMPENSATOR ZERO RES-
C       PECTIVELY.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSICN A(15),AD(15),B(15),BD(15)
      G1=6.0
      G2=6.75
      B(1)=X*Z
      B(2)=G2*P+X*Y*P+X
      B(3)=G1*P+X*R*P+G2+X*Y
      B(4)=G1+X*R+P
      B(5)=1.0
      BD(1)=Z
      BD(2)=1.0+Y*P
      BD(3)=R*P+Y
      BD(4)=R
      BD(5)=0.0
      IF(B(1).NE.0.0)GO TO 2
      DO 1 K=1,4
      A(K)=B(K+1)
    1 AD(K)=BD(K+1)
      RETURN
    2 DO 3 I=1,5
      A(I)=B(I)
    3 AD(I)=BD(I)
      RETURN
      END


      FUNCTION F(I,X,Y,Z,P,R)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(15),UD(15),A(15),AD(15)
C
C     PRCBLEM SPECIFICATICNS ARE DAMPING RATIO,NATURAL FREQUENCY,BANDWI-
C     CTH,ERROR COEFFICIENT AND ROOT SENSITIVITY.
C
      DATA N,ZE,O,B,SE,EC/4,0.5,5.0,6.0,0.2,3.2/
C
C     CALCULATION OF THE CHEBYCHEV'S FUNCTION WILL BE MADE USING AN
C     EXTERNAL SUBROUTINE
C
      CALL CHEVY(ZE,N,U,UD)
C
C     COEFFICIENTS OF THE CHARACTERISTIC EQUATION AND ITS PARTIAL DERIVA
C     TIVES WILL BE CALCULATED USING AN EXTERNAL SUBROUTINE THAT MUST BE
C     MODIFIED FOR EACH PROBLEM.
C
      CALL CALCE1(X,Y,Z,P,R,A,AD)
C
C     CALCULATION OF THE VALUES OF THE FUNCTION AT EACH STEP OF THE
C     SUBRCUTINE FOR SOLVING THE NON-LINEAR EQUATIONS.
C
      GO TO (22,23,24,25,26),I
```

```
   22 F=(A(5)*B**4-A(3)*B**2+A(1))**2+(A(2)*B-A(4)*B**3)**2-2.0*(X*Z)**2
     &-2.0*(X*B)**2
      RETURN
   23 CALL SENSIT(O,N,A,AD,U,UD,DET,DETKZ)
      F=SE*ZE*DET+X*DETKZ
      RETURN
   24 F=EC*(A(2)-X)-(X*Z)
      RETURN
   25 F=A(1)*U(1)
      DO 7 K=1,N
    7 F=F+(-1.)**K*O**K*U(K+1)*A(K+1)
      RETURN
   26 F=A(1)*U(2)
      DO 8 K=1,N
    8 F=F+(-1.)**K*O**K*U(K+2)*A(K+1)
      RETURN
      END


      FUNCTION G(I,X,Y,Z,P)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(15),A(15),UD(15),AD(15)
C
C     PROBLEM SPECIFICATIONS ARE DAMPING RATIO,NATURAL FREQUENCY,BANDWI-
C     DTH AND ROOT SENSITIVITY.
C
      DATA N,ZE,O,B,SE/4,0.5,5.0,6.0,0.2/
C
C     CALCULATION OF THE CHEBYCHEV'S FUNCTION WILL BE MADE USING AN
C     EXTERNAL SUBROUTINE
C
      CALL CHEVY(ZE,N,U,UD)
C
C     CALCULATION OF THE COEFFICIENTS OF THE CHARACTERISTIC EQUATION
C     FOR THE PARTICULAR PROBLEM.
C
      CALL CALCE1(X,Y,Z,P,0.0,A,AD)
C
C     CALCULATION OF THE VALUES OF THE FUNCTION AT EACH STEP OF THE
C     SUBROUTINE FOR SOLVING THE NON-LINEAR EQUATIONS.
C
      GO TO (22,23,24,25),I
   22 G=(A(5)*B**4-A(3)*B**2+A(1))**2+(A(2)*B-A(4)*B**3)**2-2.0*(X*Z)**2
     &-2.0*(X*B)**2
      RETURN
   23 CALL SENSIT(O,N,A,AD,U,UD,DET,DETKZ)
      G=SE*ZE*DET+X*DETKZ
      RETURN
   24 G=A(1)*U(1)
      DO 7 K=1,N
    7 G=G+(-1)**K*O**K*U(K+1)*A(K+1)
      RETURN
   25 G=A(1)*U(2)
      DO 8 K=1,N
    8 G=G+(-1.)**K*O**K*U(K+2)*A(K+1)
      RETURN
      END


      FUNCTION Q(I,X,Y,Z,P)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(15),A(15),UD(15),AD(15)
C
C     PROBLEM SPECIFICATIOS ARE  DAMPING RATIO,NATURAL FRQUENCY,BAND-
C     WIDTH AND ERROR COEFFICIENT
C
      DATA N,ZE,O,B,EC/4,0.5,5.0,6.0,3.2/
C
C     CALCULATION OF THE CHEBYCHEV'S FUNCTION WILL BE MADE USING AN
C     EXTERNAL SUBROUTINE
C
      CALL CHEVY(ZE,N,U,UD)
C
C     CALCULATION OF THE COEFFICIENTS OF THE CHARACTERISTIC EQUATION
C     FOR THE PARTICULAR PROBLEM.
C
      CALL CALCE1(X,0.0,Z,P,Y,A,AD)
C
C     CALCULATION OF THE VALUES OF THE FUNCTION AT EACH STEP OF THE
C     SUBROUTINE FOR SOLVING THE NON-LINEAR EQUATIONS.
C
      GO TO (22,23,24,25),I
   22 Q=(A(5)*B**4-A(3)*B**2+A(1))**2+(A(2)*B-A(4)*B**3)**2-2.0*(X*Z)**2
     &-2.0*(X*B)**2
      RETURN
   23 Q=EC*(A(2)-X)-X*Z
      RETURN
   24 Q=A(1)*U(1)
      DO 7 K=1,N
    7 Q=Q+(-1)**K*O**K*U(K+1)*A(K+1)
      RETURN
   25 Q=A(1)*U(2)
      DO 8 K=1,N
    8 Q=Q+(-1.)**K*O**K*U(K+2)*A(K+1)
      RETURN
      END
      FUNCTION H(I,X,Y,Z)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(15),UD(15),A(15),AD(15)
C
C     PROBEM SPECIFICATIONS ARE DAMPING RATIO,NATURAL FREQUENCY AND ROOT
C     SENSITIVITY.
C
      DATA N,ZE,C,SE/3,0.5,5.0,0.1/
C
C     CALCULATION OF THE CHEBYCHEV'S FUNCTION WILL BE MADE USING AN
C     EXTERNAL SUBROUTINE
C
      CALL CHEVY(ZE,N,U,UD)
C
C     COEFFICIENTS OF THE CHARACTERISTIC EQUATION AND ITS PARTIAL DERIVA
C     TIVES WILL BE CALCULATED USING AN EXTERNAL SUBROUTINE THAT MUST BE
C     MODIFIED FOR EACH PROBLEM.
      CALL CALCE1(X,Y,0.0,0.0,Z,A,AD)
C
C     CALCULATION OF THE VALUES OF THE FUNCTION AT EACH STEP OF THE
C     SUBROUTINE FOR SOLVING THE NON-LINEAR EQUATIONS.
C
      GO TO (22,23,24),I
   22 CALL SENSIT(O,N,A,AD,U,UD,DET,DETKZ)
      H=SE*ZE*DET+X*DETKZ
      RETURN
   23 H=A(1)*U(1)
      DO 7 K=1,N
    7 H=H+((-1.0)**K)*(O**K)*U(K+1)*A(K+1)
      RETURN
   24 H=A(1)*U(2)
      DO 8 K=1,N
    8 H=H+((-1.0)**K)*(O**K)*U(K+2)*A(K+1)
      RETURN
      END
```

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13. ABSTRACT

Algebraic method techniques are applied to design linear control systems. Computer programs to accomplish the design procedure are presented. Several combinations of feedback and cascade compensated systems using different specifications are studied. Specifications considered include the system damping ratio, undamped natural frequency, bandwidth, steady state error coefficients and root sensitivity. The method is shown to be applicable to any transfer function and capable of working with any number of parameters.

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